Analysis of Multi Dielectric Layer Structures in SDM by Iterative Green's Function Calculation Technique

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Abstract : The Spectral Domain Method is widely applied to analyse two layer planar microwave circuits. Adaptation of this technique to multilayer structures requires efficient Green's function derivation. This paper provides this derivation for computer design tool programmers.

1 Introduction

Planar microwave circuits and planar antennas are essential components of electronic communication systems utilising the microwave region of electromagnetic spectrum. Multilayer structures are often used in the design of microwave components in order to allow for more versatile design with higher density. Although most of the early analysis techniques are based on either *quasi*–static or magnetic wall approximations, recent applications require *full–wave* techniques because of high operating frequency and of high density.

The Spectral Domain Method is one of the full–wave techniques and is widely applied to the analysis of planar microwave circuits. SDM is a frequency domain technique and convenient form of the Green's function is available in the spectral domain. But in the case of multilayer structure analysis, it is very important to find an efficient way to derive Green's function. This derivation must be suitable for computer programming. The Green's function can directly be derived by solving Maxwell's equation in the spectral domain with suitable boundary condition but extending this procedure for multiple dielectric layers becomes too complicated. A generalised spectral domain Green's function for multilayer dielectric substrates is derived by Das [1] in terms of suitable components of vector electric and magnetic potential. With these vector potentials, the boundary conditions were simplified into equivalent transmission lines problems. Despite this simplification, his technique is found by the author to be still complicated and not favourable for computer programming.

The iterative calculation technique has been developed by using the immitance approach introduced by Itoh [2]. In this technique a transmission line is assumed to be terminated by another transmission line of different characteristic impedance. In the derivation, the conventional transmission line theory is used to find the characteristic impedance of the corresponding layer. The Asymptotic forms of th Green's function which is proven to be effective for single layer structures is derived for the circuits with multiple dielectric layer.

2 Derivation of Dyadic Green's Function

Although the method may be applied to other printed circuit structures, a simple microstrip resonator shown in figure 1 is used for the formulation.

The basic concept of the immitance approach can be understood if the inverse transform of the Fourier transform in equation 1 for the field is examined.

$$\phi(x,z) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi(k_x,k_z)e^{-j(k_x x + k_z z)}dk_x dk_z \tag{1}$$

From this expression, all field components are a superposition over k_x and k_z of inhomogeneous (in y) plane waves which are propagating in the direction of θ from the z–axis , where $\theta = \cos^{-1}\left(\frac{k_z}{\sqrt{k_x^2+k_z^2}}\right)$. For each θ, waves

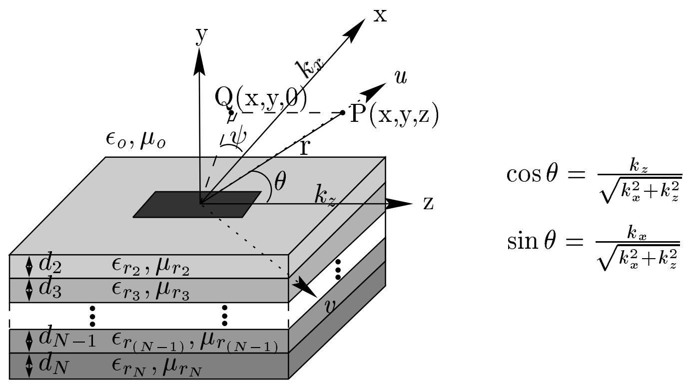

Figure 1: Cross–section of a multilayered microstrip resonator

may be decomposed into TM–to–y ($\mathbf{E}_y, \mathbf{E}_v, \mathbf{H}_u$), and TE–to–y ($\mathbf{H}_y, \mathbf{H}_v, \mathbf{E}_u$) where the coordinates v and u are as shown in figure 1 and related with (x, z) via

$$u = z\sin\theta - x\cos\theta \qquad v = z\cos\theta + x\sin\theta \tag{2}$$

The current $\mathbf{J}_v$ creates only the TM fields, because it is concerned with $\mathbf{H}_u$ and likewise $\mathbf{J}_u$ creates the TE fields. Therefore, an equivalent circuit for the TM and TE fields can be drawn, as shown in figure 2. The characteristic impedance in each region is given by;

$$\mathbf{Z}_{TM_i} = \frac{\mathbf{E}_v}{\mathbf{H}_u} = \frac{\gamma_i}{j\omega\epsilon_i}, \qquad \mathbf{Z}_{TE_i} = -\frac{\mathbf{E}_u}{\mathbf{H}_v} = \frac{j\omega\mu_i}{\gamma_i} \tag{3}$$

where $\gamma_i = \sqrt{k_x^2 + k_z^2 - k_i^2}$, $i = 1..N$ and $k_i^2 = \omega^2\mu_i\epsilon_i$. The γ_i is the propagation constant in the y direction in the i^{th} region. All boundary conditions for the TM and TE waves are incorporated in the equivalent circuits. The electric fields $\mathbf{E}_v$ and $\mathbf{E}_u$ are continuous at $y = 0$ and given by;

$$\mathbf{E}_v(k_x, k_z, 0) = \mathbf{Z}_e(k_x, k_z, 0)\mathbf{J}_v(k_x, k_z, 0), \qquad \mathbf{E}_u(k_x, k_z, 0) = \mathbf{Z}_h(k_x, k_z, 0)\mathbf{J}_u(k_x, k_z, 0) \tag{4}$$

$\mathbf{Z}_e$ and $\mathbf{Z}_h$ are the input impedances looking into the equivalent circuits at $y = 0$ and are given by;

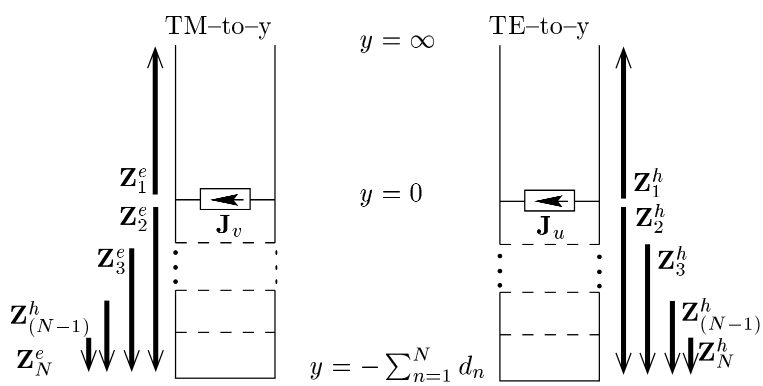

Figure 2: Equivalent transmission lines for the TM and TE fields

$$\mathbf{Z}_e(k_x, k_z, d) = \left(\frac{1}{\mathbf{Z}_1^e} + \frac{1}{\mathbf{Z}_2^e}\right)^{-1}, \qquad \mathbf{Z}_h(k_x, k_z, d) = \left(\frac{1}{\mathbf{Z}_1^h} + \frac{1}{\mathbf{Z}_2^h}\right)^{-1} \tag{5}$$

where $\mathbf{Z}_1^e$ and $\mathbf{Z}_2^e$ are input impedances looking into the corresponding regions at $y = 0$ in the TM equivalent circuit, whereas $\mathbf{Z}_1^h$ and $\mathbf{Z}_2^h$ are those impedances in the TE circuit. The definition of $\mathbf{Z}_1^e$ and $\mathbf{Z}_1^h$ are $\mathbf{Z}_{TM_1}$ and $\mathbf{Z}_1^h = \mathbf{Z}_{TE_1}$ respectively. As seen in figure 2, a transmission line is terminated by another transmission line of different characteristic impedance. Therefore conventional transmission line theory can be used to find the $\mathbf{Z}_2^e$ and $\mathbf{Z}_2^h$ as

$$\begin{aligned}
\mathbf{Z}_N^e &= \frac{\mathbf{Z}_{TM_N}}{\coth \gamma_N d_N} & \mathbf{Z}_N^h &= \frac{\mathbf{Z}_{TE_N}}{\coth \gamma_N d_N} \\
\mathbf{Z}_{N-1}^e &= \mathbf{Z}_{TM_{(N-1)}} \frac{\mathbf{Z}_N^e \coth \gamma_{N-1} d_{N-1} + \mathbf{Z}_{TM_{(N-1)}}}{\mathbf{Z}_{TM_{(N-1)}} \coth \gamma_{N-1} d_{N-1} + \mathbf{Z}_N^e} & \mathbf{Z}_{N-1}^h &= \mathbf{Z}_{TE_{(N-1)}} \frac{\mathbf{Z}_N^h \coth \gamma_{N-1} d_{N-1} + \mathbf{Z}_{TE_{(N-1)}}}{\mathbf{Z}_{TE_{(N-1)}} \coth \gamma_{N-1} d_{N-1} + \mathbf{Z}_N^h} \\
\vdots &= \vdots & \vdots &= \vdots \\
\mathbf{Z}_2^e &= \mathbf{Z}_{TM_2} \frac{\mathbf{Z}_3^e \coth \gamma_2 d_2 + \mathbf{Z}_{TM_2}}{\mathbf{Z}_{TM_2} \coth \gamma_2 d_2 + \mathbf{Z}_3^e} & \mathbf{Z}_2^h &= \mathbf{Z}_{TE_2} \frac{\mathbf{Z}_3^h \coth \gamma_2 d_2 + \mathbf{Z}_{TE_2}}{\mathbf{Z}_{TE_2} \coth \gamma_2 d_2 + \mathbf{Z}_3^h}
\end{aligned} \tag{6}$$

The final part of the immitance approach formulation is to map from the (u, v) to (x, z) co–ordinate system for the spectral wave corresponding to each θ given by k_x and k_z. Because of the coordinate transform in equation 2, $\mathbf{E}_x$ and $\mathbf{E}_z$ are linear combination of $\mathbf{E}_u$ and $\mathbf{E}_v$. Similarly $\mathbf{J}_x$ and $\mathbf{J}_z$ are superpositions of $\mathbf{J}_u$ and $\mathbf{J}_v$. When the above notations are applied, the Green's function elements are found to be:

$$\mathbf{G}_{zz} = \frac{k_z^2}{k_x^2 + k_z^2}\mathbf{Z}_e + \frac{k_x^2}{k_x^2 + k_z^2}\mathbf{Z}_h \qquad \mathbf{G}_{xx} = \frac{k_x^2}{k_x^2 + k_z^2}\mathbf{Z}_e + \frac{k_z^2}{k_x^2 + k_z^2}\mathbf{Z}_h \tag{7}$$

$$\mathbf{G}_{zx} = \frac{k_x k_z}{k_x^2 + k_z^2}(-\mathbf{Z}_e + \mathbf{Z}_h) \qquad \mathbf{G}_{xz} = \mathbf{G}_{zx} \tag{8}$$

The dyadic Green's function for multilayer structure is computationally complex and this complexity increases with the number of layers. There is however a possibility to use the asymptotic form of the Green's function. For large transform variables (k_x and k_z), the original Green's function for a multilayer structure converges to the Green's function of a simple two layer structure. For this, they must satisfy the conditions which are given by;

$$\gamma_i = \sqrt{k_x^2 + k_z^2 - k_{max}^2} \approx \sqrt{k_x^2 + k_z^2}, \qquad Coth\gamma_{ij} d_{min} \approx 1 \tag{9}$$

where $k_{max} = \max(k_i)$ and $d_{min} = \min(h_i)$.

So, for the large Fourier transform variables, all multilayer structures are equivalent to the structure containing the immediate two layers, extended to infinity, on each side of the strip. The reason for this is that large values of transform variables account for the reactive field of the source, which is a localised effect.

With the above argument, the asymptotic form of the Green's function of multilayered structures can be directly obtained from the asymptotic form of the Green's function for two layer structure

3 Conclusion

This paper presented an iterative solution to derive the Green's function for the multi dielectric layer structures. This derivation is very easy to apply to computer programming. The derivation of the Asymptotic Form of the Green's function was also mentioned along with two key conditions.

References

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