Efficient Numerical Integration of the Impedance Matrix for the Analysis of Open Planar Circuits and Antennas

Hasan H. Balik
Dept. of Elect. & Electronics Eng., Karadeniz Tech. Univ., Trabzon, TURKEY
C. J. Railton
Dept. of Elect. & Electronics Eng., Univ. Bristol, Bristol, UK

Abstract : The Spectral Domain Method is a very fast and powerful technique to analyse planar microwave circuits. But analysis of complex circuits by SDM requires more computer resources. The benefit of defining the current basis function as two separable function is exploited to improve the run–time and memory requirements.

1 Introduction

Passive planar microwave circuits and planar antennas are essential components of electronic communication systems utilising the microwave region of electromagnetic spectrum. As the frequency increased up to unused frequency bands with more available bandwidth, the simplified circuit theory, which is valid at low frequencies, can not simply be employed for the analysis of the microwave circuits of interest. Therefore, it is necessary to use full–wave numerical techniques and the Spectral Domain Method, which is widely used in the literature, has been chosen.

The first requirement in SDM for the analysis of open structures is the definition of the unknown current distribution on the metallisation. Therefore the first step of the analysis is to expand the unknown surface current as a set of known basis functions with unknown coefficients. It has been shown and commonly used that a rooftop function as a current basis function allows the unknown surface current on an irregular shaped metallisation to be defined. But this approach results in a large numbers of basis functions for convergence. Balik has defined the rooftop function as a function of its location in [1] allowing subgridding. With this definition the discontinuity has modelled by fine rooftop functions whereas course rooftops are used where only slow changes occur in current distribution. The pre–calculated basis function as a linear combination of the rooftop function as well as a current wave function with a pre–calculated wave number was also defined in [1].

The SDM is a frequency domain technique and requires repeated calculation of the impedance matrix at each frequency point of interest. In [2], the adaptive integration technique which consists of an adaptive truncation and adaptive integration step has been introduced to limit the two–dimensional continuous numerical integration over an infinite surface because of having open structure to finite computer resources. In addition, the asymptotic form of the Green's function, which is defined for large values of transform variables, is used to calculate the impedance matrix. The symmetry in Green function has also been exploited to reduce the run–time required for the impedance matrix calculation.

The enhancement introduced here is effective on the first and second requirement in SDM analysis. It is developed to reduce the number of current basis function required in one dimension to speed up overall two dimensional numerical integration. This is very effective and could give %99 improvement in most cases.

2 Method of Calculation

As mentioned previously, the unknown current distribution is expanded as a set of known basis functions with unknown coefficients. The known current basis functions can be either sub–domain or pre–calculated functions. Both functions are defined as two separate functions for each direction as:

$$\mathbf{J}_s(k_x,k_z) = \sum_{n=1}^{N} a_{sn}\mathbf{J}_{sn}(k_x,k_z) = a_{sn}\left(\sum_{nx=1}^{N_x}\mathbf{J}_{snx}(k_x)\sum_{nz=1}^{N_z}\mathbf{J}_{snz}(k_z)\right) \qquad n = 1..N \tag{1}$$

and the Method of Moments requires a set of weighting functions, which is identical to the set of current basis functions if the procedure is Galerkin's, is defined by:

$$\mathbf{w}_t(k_x,k_z) = \sum_{n=1}^{N}\mathbf{w}_{tn}(k_x,k_z) = \sum_{nx=1}^{N_x}\mathbf{w}_{tnx}(k_x)\sum_{nz=1}^{N_z}\mathbf{w}_{snz}(k_z) \tag{2}$$

where $N = N_xN_z$. The calculation of the impedance matrix elements is given by;

$$\mathbf{Z}_{st}(n,m) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\sum_{m=1}^{N}\mathbf{w}_{tm}(k_x,k_z)\right)\mathbf{G}(k_x,k_z,d,\omega)\left(\sum_{n=1}^{N}\mathbf{J}_{sn}(k_x,k_z)\right)dk_xdk_z \tag{3}$$

With reference to equations 1 and 2, equation 3 can be rearranged as;

$$\begin{aligned}\mathbf{Z}_{st}(n,m) = \int_{-\infty}^{\infty}&\left[\int_{-\infty}^{\infty}\left(\sum_{mx=1}^{N_x}\mathbf{w}_{tmx}(k_x)\right)\mathbf{G}(k_x,k_z,d,\omega)\left(\sum_{nx=1}^{N_x}\mathbf{J}_{snx}(k_x)\right)dk_x\right]\\&\left(\sum_{mz=1}^{N_z}\mathbf{w}_{tmz}(k_z)\right)\left(\sum_{nz=1}^{N_z}\mathbf{J}_{snz}(k_z)\right)dk_z\end{aligned} \tag{4}$$

The integration thus splits into two dependent integrations, but the number of integrands is independent for each integration in one dimension. The run–time for the impedance matrix calculation is a function of the number of current basis functions used in both direction and can be formulated as;

$$\frac{N_z(2N_xN_z+1)}{(2N_x+1)} \tag{5}$$

As seen in equation in 5, the run–time required is problem dependent. If pre–calculated current basis functions are used, the savings in computer memory and run–rime are even greater. In the author's experience, this enhancement gives around 99% saving for the one dimensional integration and 90% for overall integration on most analysis. It must be noted that, as the current basis functions are defined as two separable functions in both directions, the values of current basis functions at each integral point are only calculated once and used during numerical integration for the entire frequency range.

3 Analysis of Simple Low–Pass Filter

Measurement results and filter dimensions are available for the microstrip low–pass filter in [3]. This filter is analysed in order to demonstrate the improvement gained by the enhancements which is introduced in this paper and to compare the calculated results with available measurement data. It must be noted that overlapping rooftop functions are used as current basis function to model the unknown current distribution of the metallisation of the filter. The dimensions of the rooftop functions are 0.60325 mm for the feedlines and 0.635 mm for the central strip. The total number of rooftop functions as current basis functions is 594 (297 in x and 297 in z components). As given in equation 1, the current basis functions are two separable functions. and 74 x–directed basis functions are used to derive 594 rooftop functions in both dimensions. With this definition, 176715 impedance matrix elements are derived from 2275 sub–impedance matrix elements. Therefore the impedance matrix elements needed to be integrated are reduced by 99.98% in one dimension. The S–parameter results are plotted in figures 1. As seen in figure 1, the calculated results and measurements are in very good agreement.

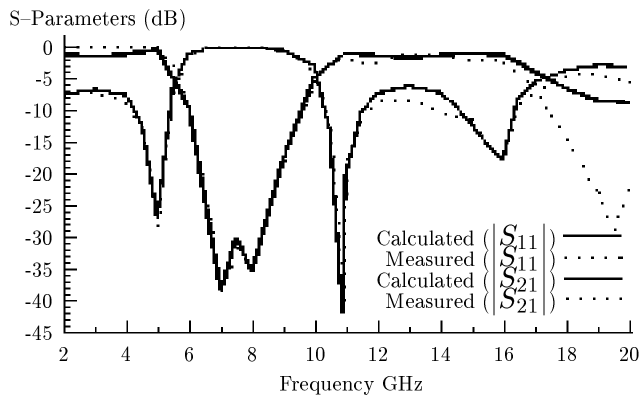

Figure 1: Plot of S–parameters' magnitude for the low–pass filter

4 Conclusion

The feature of current basis function which is to be two separable function in each dimension is exploited in this paper to speed up impedance matrix calculation that is required at each frequency point. More than 99% improvement is gained without loosing the generality of the technique in most practical problems. The low–pass filter is analysed and results are compared with measured data.

References

[1] H. H. Balik and C. J. Railton, "Sub–gridding in the spectral domain method for the analysis of planar circuits and antennas," in *3rd International conference on telecommunications in modern satellite, cable and broadcasting services*, pp. 592–595, October 1997.

[2] H. H. Balik and C. J. Railton, "Adaptive numerical integration technique for the analysis of open planar circuits and antennas," in *3rd High Frequency Postgraduate Student Colluquium*, pp. 100–105, September 1997.

[3] D. M. Sheen, S. M. Ali, M. D. Abdouzahra, and J. A. Kong, "Application of the three–dimensional finite-difference time–domain method of the analysis of planar microstrip circuits," *IEEE Transaction on Microwave Theory and Technique*, vol. 38, pp. 849–857, July 1990.

INTERNATIONAL SYMPOSIUM ON ELECTROMAGNETIC THEORY

U.R.S.I.
Commission B

INTERNATIONAL UNION OF RADIO SCIENCE

ARISTOTLE UNIVERSITY OF THESSALONIKI

PROCEEDINGS

25 - 28 MAY 1998 - THESSALONIKI GREECE