Efficient Enhancements in Spectral Domain Method To Speed up Open Planar Circuit Analysis

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Abstract

The Spectral Domain Method is a very fast and powerful technique to analyse planar microwave circuits. But available techniques for simulating the excitation of open planar microwave circuits are not very effective at relatively low frequencies. In this contribution new compensation functions are introduced which correctly model the excitation over the whole microwave frequency region.

1 Introduction

For the last few decades, designers of microwave and millimetre–wave integrated circuits have come depend heavily on computer–aided techniques to reduce design time and improve performance. This is because most of the practical electromagnetic problems can be solved numerically and cannot be solved analytically. Recently as the complexity, component density and operating frequencies of these circuits have increased dramatically, the problems posed to the designer of CAD tools have become much more difficult. Moreover the rapid development and availability of increasing computing power do not solve the problem. The complexity of the circuit to be designed and computational overhead required for accurate modelling will outstretch any development in hardware.

Most of the early analysis methods for microwave integrated circuits were based on either *quasi*–static or magnetic wall approximation which are only valid at low frequencies. The use of high operating frequencies make such approximations insufficiently accurate. At high frequencies it is necessary to use *full–wave* analysis to get accurate results. In this context the descriptive term *full–wave* is used to define the set of numerical techniques which include all field components and thus allow rigorous analysis of dispersive structures where coupling between circuits elements cannot be ignored in the microwave frequency band of interest.

The Spectral Domain Method which is based on solving coupled integral equation [1, chapter 3 and 5] is one of the widely used *full–wave* analysis techniques. It is, however, limited by the requirement for a large amount of computer resources. Notwithstanding the rapid increase in the availability of the computer power, work is still necessary to increase the efficiency of the algorithm used.

The first requirement for the analysis of open structures in SDM is the definition of the unknown current distribution on the metallisation. Therefore the first step of the analysis is to expand the unknown surface current as a set of known basis functions with unknown coefficients. It has been shown and commonly used that a rooftop function [2,3] as a current basis function allows the unknown surface current on an irregular shaped metallisation to be defined. But this approach results in a large numbers of basis functions for convergence and the rooftop functions must be defined identical apart from a shift from the origin [2,3]. The use of pre–calculated basis functions reduces the impedance matrix to be calculated, but pre–calculated basis functions must be a combination of rooftop functions [4] to exploit the FFT. Balik has defined the rooftop function as a function of its location in [5] allowing subgridding. With this definition the discontinuity has modelled by fine rooftop functions whereas course rooftops are used where only slow changes occur in current distribution. The pre–calculated basis function as a linear combination of the rooftop function as well as a current wave function with a pre–calculated wave number was also defined in [5]. These enhancements remedy the limitations given in [2,3]

The SDM is a frequency domain technique and requires repeated calculation of the impedance matrix at each frequency point of interest. In [6], the adaptive integration technique which consists of an adaptive truncation and adaptive integration step has been introduced to limit the two–dimensional continuous numerical integration over an infinite surface because of having open structure to finite computer resources. The location vector calculation, which reduces the number of impedance matrix elements, is introduced and used in [7] in the region where identical sub–domain basis functions used. For this region, a sub–domain basis function is divided into two parts. First one is identical for each function and second one represents its location. By this enhancement, N(N+1)/2 number of impedance matrix elements calculated are reduced to 3N, where N is the size of the impedance matrix. In addition, the asymptotic form of the Green's function, which is defined for large values of transform variables, is used to calculate the impedance matrix. The symmetry in Green function has also been exploited to reduce the run–time required for the impedance matrix calculation.

The enhancement introduced in [8] is effective on the first and second requirement in SDM analysis. It is proposed by Balik to reduce the number of current basis function required in one dimension to speed up overall two dimensional numerical integration. This is very effective and could give up–to %99 improvement.

The circuits are presented by N–port network parameters. Scattering parameters, otherwise called S–parameters are utilised but can be transformed to other representations by using standard formulae. This paper highlights the lack of information, drawbacks of available excitation modelling techniques for the derivation of the S–parameter of the open planar circuits and introduces a novel excitation modelling for efficient, fast accurate analysis in SDM.

2 Available Excitation Modellings

The details available in the literature for the extraction of the S–parameters of planar circuits using the SDM or other related techniques are limited in scope as the topic is avoided or ignored in most papers. For shielded planar microwave circuits, the S–parameters at a spot frequency are derived in [9], [10, chapter5] from the solution of a matrix equation. In these implementations, the tangential electric field is assumed to be identically zero because of the existence of side–walls. Since there do not exist in open structures a different method must be sought. In addition, the effects of the port/feed arrangement is assumed to be negligible. The extraction of the S–parameters from the knowledge of the surface current distribution is achieved by applying transmission line theory to the feedlines (de–embedding algorithm) [11,12], but this method can only be used when the line length is bigger than a half wavelength. A method was introduced by Jackson [13] to calculate S–parameters of gap–discontinuities in 1985 for open planar circuits, but his technique, in contrast to the method described here, is not complete and efficient at relatively low frequencies. This is because the cosine portion of the travelling wave is truncated one–quarter of a guide wavelength from a zero of the sine. The length of the truncated portion is a function of the operating frequency. At low frequencies, its length becomes larger than the entire circuit's dimension and a large number of extra rooftop functions are required in order to avoid spurious numerical reflections.

3 Improved Technique for Excitation Modelling

For the formulation described in this paper, all circuits are considered to be connected to semi–infinite feedline, whose width is identical to the joining port, at each port. The fundamental microstrip mode is assumed to propagate on the feedlines and thus the travelling current waves are chosen as current basis functions given in equation 1.

$$
\begin{aligned}
J_i(z) &= \begin{cases} e^{-jk_n(z-z_i)} & -L+z_i \le z \le z_i \\ 0 & \text{otherwise} \end{cases} \\
J_r(z) &= \begin{cases} -a_r e^{jk_n(z-z_i)} & -L+z_i \le z \le z_i \\ 0 & \text{otherwise} \end{cases} \\
J_t(z) &= \begin{cases} a_t e^{-jk_n(z-z_o)} & z_o \le z \le L+z_o \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
\tag{1}
$$

Here k_n is the pre–calculated wavenumber of the feedline, L is the length of the feedline and z_s $(s = i, o)$ is the offset of the port from the origin. The letters i, t, r indicate the incident, transmitted and reflected current waves respectively and the unknown coefficients a_r, a_t are coefficients of the reflected and transmitted current waves which are to be calculated. For the excited port, the current is;

$$J_{input} = J_i + J_r \tag{2}$$

and for the ports which are not excited, the current is;

$$J_{output} = J_t \tag{3}$$

The Fourier transforms of the current basis functions for the feedlines as defined in equation 1 are given by;

$$\mathbf{J}_i(k_n, k_z) = \frac{2}{k_z - k_n} \sin\left((k_z - k_n)\frac{L}{2}\right) e^{-j(k_z - k_n)\frac{L}{2}} e^{jk_z z_i}$$
$$\mathbf{J}_r(k_n, k_z) = \frac{2}{k_z + k_n} \sin\left((k_z + k_n)\frac{L}{2}\right) e^{-j(k_z + k_n)\frac{L}{2}} e^{jk_z z_i}$$
$$\mathbf{J}_t(k_n, k_z) = \frac{2}{k_z - k_n} \sin\left((k_z - k_n)\frac{L}{2}\right) e^{j(k_z - k_n)\frac{L}{2}} e^{jk_z z_o}$$

L is theoretically extends to infinity, but in practice is chosen to be an integer number of half wavelengths [3, 13]. As shown in equations 2, 3 and in figure 1, the current basis functions of the feedlines have a real ($\cos k_n z$) and an imaginary ($\sin k_n z$) part. The real part causes current discontinuities in the direction of current flow, because the triangle function as a rooftop function component is identically zero at the interface between the port and the adjacent feedline, whereas $\cos(k_n z)$ has a finite value. The problem is the same in the direction perpendicular to current flow, because in this case the step function as a rooftop function component has a finite value, whereas $\sin(k_n z)$ is identically zero. Therefore extra basis functions are still required at each interface between the port and adjacent feedline for an accurate solution. These functions are introduced and referred to by the author as *compensation* functions which are shown in figure 2. The compensation functions are defined for the input port in the space domain by:

$$J_z(z) = \begin{cases} 1 - \frac{z - z_i}{l_z} & z_i \leq z \leq z_i + l_z \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

$$J_x(z) = \begin{cases} \frac{z - z_i}{l_z} & z_i \leq z \leq z_i + l_z \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

and for the output port;

$$J_z(z) = \begin{cases} 1 + \frac{z - z_o}{l_z} & z_o - l_z \leq z \leq z_o \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

$$J_x(z) = \begin{cases} -\frac{z - z_o}{l_z} & z_o - l_z \leq z \leq z_o \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where l_z is the size of the compensation functions in the direction of propagation.

$J_z(z)$ in equation 4 and figure 2(a) transfers the effect of the cosine portion of the incident current wave in the direction of current flow, whereas $J_x(z)$ in equation 5 and figure 2(a) transfers the effect of the sine portion of the incident current wave in the direction perpendicular to the current flow. Similarly $J_z(z)$ in equation 6 and figure 2(b) transfers the current wave to the output port in the direction of propagation whereas $J_x(z)$ in equation 7 and figure 2(b) transfers the current wave in the transverse direction.

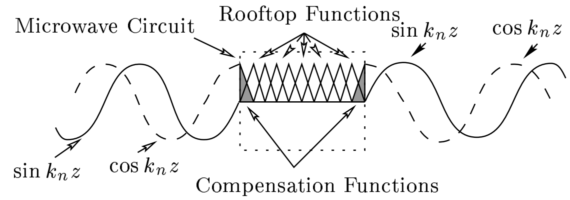
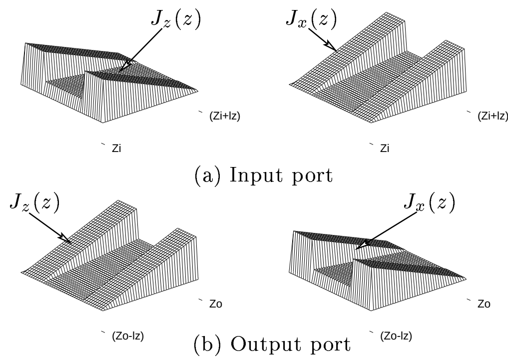


Figure 1: Illustration of the excitation in the direction of current flow



(a) Input port

(b) Output port

Figure 2: Compensation functions for input and output ports

As illustrated in figure 2, the compensation function is actually a semi–rooftop function with an offset from the origin in either $-z$ or $+z$ directions and the missing portion of the rooftop function has been completed by the current waves. Fourier transforms of the half rooftop functions are given by;

$$\mathbf{J}_{left}(k_z) = \frac{1}{k_z^2 l_z}[(1 - \cos(k_z l_z)) + j(\sin(k_z l_z) - k_z l_z)]$$
$$\mathbf{J}_{right}(k_z) = \frac{1}{k_z^2 l_z}[(1 - \cos(k_z l_z)) - j(\sin(k_z l_z) - k_z l_z)]$$

where $\mathbf{J}_{left}$ and $\mathbf{J}_{right}$ indicate the Fourier transform of the left and right hand side of a rooftop function. The derivation of the compensation functions from the left and right hand side of the rooftop function for the input port are given by;

$$\mathbf{J}_z(k_z) = \mathbf{J}_{right}(k_z) e^{jk_z z_{off}} \tag{8}$$
$$\mathbf{J}_x(k_z) = \mathbf{J}_{left}(k_z) e^{jk_z (z_{off} + l_z)} \tag{9}$$

and for the output port, they are given by;

$$\mathbf{J}_z(k_z) = \mathbf{J}_{left}(k_z)e^{jk_z z_{off}} \tag{10}$$

$$\mathbf{J}_x(k_z) = \mathbf{J}_{right}(k_z)e^{jk_z(z_{off}-l_z)} \tag{11}$$

Including all current basis functions of the entire system, the total current is expressed as:

$$\mathbf{J}_{total} = \mathbf{J}_{box}(k_x, k_z) + \sum_{n=1}^{N}(a_{pn}\mathbf{J}_{port_n}(k_x, k_n k_z) + a_{cn}\mathbf{J}_{comp_n}(k_x, k_z))$$

where N is the number of ports, $\mathbf{J}_{box}$ refers to the basis functions of the microwave circuit, $\mathbf{J}_{port_n}$ is either the sum of the incident and reflected current wave for the excited port or the transmitted current wave for unexcited ports and $\mathbf{J}_{comp_n}$ is the Fourier transform of the compensation function. To calculate the S–parameters of the circuit, a_{pn} must be known. The Method of Moments is employed to eliminate the electric field components and to find the unknown coefficients. A total of N weighting functions must be defined to complete the algorithm. These are chosen to be a triangle function which straddles the lines separating each port and the feedline in the direction of current flow and a pre–calculated basis function in the direction perpendicular to current flow. After application of the Method of Moments, the matrix equation yields,

$$[\mathbf{Z}][\mathbf{I}] = [0] \tag{12}$$

Because two travelling waves, which are the incident current wave with unit amplitude and the reflected current wave with unknown amplitude, are used for the feedline connected to the input port of the circuit, $\mathbf{Z}$ is not a square matrix and contains unknown coefficients, as well as the known unit amplitude. The column of the impedance matrix corresponding to the incident current wave products is moved to the right hand side of the equation and the equation 12 becomes,

$$[\mathbf{Z}]_n[\mathbf{I}]_n = [\mathbf{Z}]_i \tag{13}$$

where $\mathbf{Z}_n$ consists of the elements related to unknown current basis functions and $\mathbf{Z}_i$ is a column vector containing the elements related to incident current waves. Any root–finding procedure can be applied to solve equation 13 and the unknown coefficients can be found. The S–parameters of the N–port circuit are actually the coefficients of the reflected and the transmitted current waves.

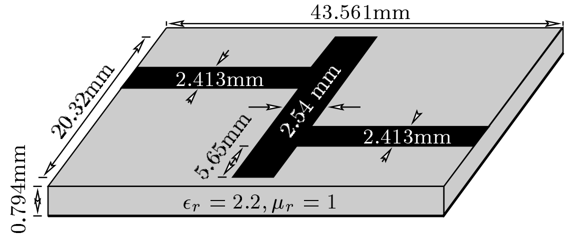


Figure 3: Low–pass filter detail

4 Numerical Results

It must be noted that, Jackson's excitation technique is used with no extra rooftop functions to complete the truncated portion of the current wave. This is because of using identical number of basis functions in each model and illustrating the improvement in the accuracy for identical number of basis functions.

4.1 Simple Low–pass Filter

Measurement results are available for the microstrip low–pass filter [14] shown in figure 3. This filter is analysed to demonstrate the total improvement in this implementation and to compare the calculated results to available measurement data. The dimensions and parameters of the dielectric substrate are given in figure 3.

It is assumed that the truncated current wave and the extra set of rooftop functions to complete this truncated portion are used to excite the open circuit of interest (Jackson's technique). The minimum operating frequency is 2 GHz and the corresponding length of the cosine is 27.42 mm, which is 45 times bigger than the size of the rooftop which is 0.60325 mm in this analysis. For the maximum operating frequency of 20 GHz, the corresponding length of truncation is 2.6925 mm, which is still 4 times bigger than the rooftop. To complete the missing portion in order to get accurate results, 528 extra rooftop functions are required. In contrast to this, with the use of the compensation functions, 4 functions are sufficient. The S–parameter results are plotted in figures 4 where it can be seen that the calculated results and measurements are in very good agreement.

5 Conclusion

We have shown that realistically complex microstrip circuits can be rigorously analysed at relatively low frequencies using new compensation functions which are defined at the interface between a port and a feedline in the Spectral Domain Method. The compensation functions are shown to be effective to

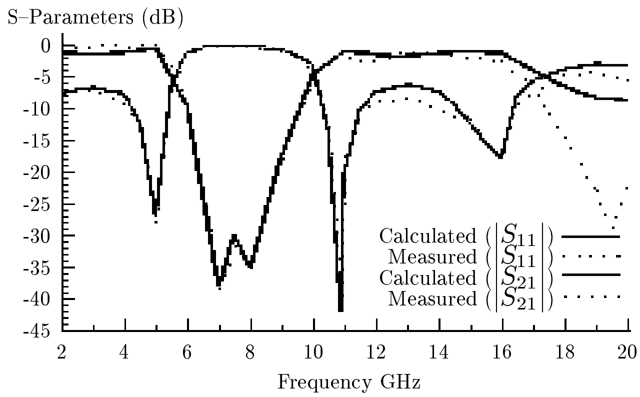


Figure 4: Plot of S–parameters' magnitude for the low–pass filter

model the effect of the excitation of the circuit. The results compare well with published measurements.

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