Sub–gridding in the Spectral Domain Method for the Analysis of Planar Circuits and Antennas

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Abstract— **Enhancements to the Spectral Domain Method for open planar microwave circuits and antennas are presented which reduce the number of current basis functions required. The technique which exploits the benefits of using the combination of sub–gridding and pre–calculated basis functions is introduced and numerical results are given for an example structure to verify the accuracy of the technique.**

Keywords— **SDM, Spectral Domain Method,open planar circuits, planar antennas, sub–gridding, pre–calculated**

I. Introduction

For the last two decades, the high–speed computer has influenced the computation of electromagnetic problems, to the point that most practical computations of fields are now done numerically on computers. This is because most of the practical problems in the electromagnetics can be solved numerically but cannot be done analytically. Therefore computers are necessary for numerical solutions. As a result the science of computational electromagnetics is a mixture of electromagnetic theory, mathematics and numerical analysis.

The requirement to predict the behaviour of planar microwave components and circuits has existed for many years and designers have come depend heavily on computer–aided–design techniques to reduce design time and improve performance. Recently, however, as the complexity, component density and operating frequencies of the circuits have increased dramatically, the problems have become even more difficult. It is inadequate to treat a circuit as a set of isolated components or to use models based on *quasi–static* approximation at high frequencies and, hence, necessary to use *full–wave* analysis to get accurate results.

Currently a number of methods which fulfill this requirement are available. These are the Finite Difference Time Domain (FDTD) [1], the Transmission Line Matrix (TLM) [2], the Integral Equation Method (IEM) [3, 4] and the Spectral Domain Method (SDM) [5, 6]. The first two techniques are numerical methods with minimal analytical preprocessing and thus suited to a wide range of applications with a basic implementation but tend to need a large amount of computer resources. The Integral Equation Method and the Spectral Domain Method are both based on solving coupled integral equations but calculations take place in different domains. The majority of the calculation in the IEM take place in the space domain where no convenient form of the Green's function exists for the planar structures of interest. It is therefore common practice to use the spectral domain form of the Green's function and then use the inverse transform to numerically return to the space domain. This procedure is computationally intensive and thus is one of the major disadvantages of the method.

The chosen technique for this contribution is the Spectral Domain Method, because the SDM needs relatively large overheads in analytical preprocessing but this results in efficient implementations for the planar structures of interest. The main advantage of SDM is to reduce the coupled integral equation to a simpler set of algebraic equations by taking Fourier transform. Moreover for the planar circuits of interest in this paper a convenient form of the Green's function exists in the spectral domain.

Analysis of complex planar circuits by the SDM requires the definition of the unknown current distribution on the metalisation of the circuit. Therefore the first step of the analysis is to expand the unknown surface current distribution as a set of known basis functions with unknown coefficients, after that the Method of Moments [7, chapter5] is applied to find these coefficients.

The choice of basis function is crucial to the efficiency of the technique and special care must be taken to approximate the unknown current distribution as closely as possible otherwise a large number of basis functions are required for convergence. It has been shown and commonly used that a rooftop function [6, 8] as a current basis function allows the unknown surface current on an irregular shaped metalisation to be defined. But this approach results in a large numbers of basis functions for convergence and the

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rooftop functions must be defined identical apart from a shift from the origin [6,8]. The use of pre–calculated basis functions reduces the impedance matrix to be calculated, but pre–calculated basis functions must be a combination of rooftop functions [5] to exploit the FFT.

In this paper a technique is described which is capable of characterising microwave circuits using only small desk–top computer. The technique which is based on the Spectral Domain Method exploits the benefits of using pre–calculated current basis functions. In addition the size of the rooftop basis function is allowed to vary throughout the problem space so that areas of rapid changes in current distribution are more accurately modelled. This novel sub–gridding approach leads to improvements in computational efficiently.

II. Current Basis Functions

The definition of known basis functions which are valid on the metalisation of the circuit in the space domain and of which analytical Fourier transform exist are required for the Method of Moments solution. The basis functions must be chosen to approximate the unknown current distribution of the metalisation as closely as possible. If they are not chosen carefully then a large number of basis functions are required for convergence. Therefore the choice of current basis functions is crucial to the efficiency of the technique.

A. Rooftop Basis Functions

Arbitrary shaped planar microwave circuits have been modelled using rooftop functions as sub–domain basis functions by numerous authors [5, 6, 8]. The geometry of rooftop function is illustrated in figure 1 and is defined in space domain by :

$$J_{xn}(x,z) = \begin{cases} 1 - \frac{|x-x_n|}{l_x} & \begin{matrix} x_n - l_x \le x \le x_n + l_x \\ z_n - l_z \le z \le z_n + l_z \end{matrix} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$J_{zn}(x,z) = \begin{cases} 1 - \frac{|z-z_n|}{l_z} & \begin{matrix} x_n - l_x \le x \le x_n + l_x \\ z_n - l_z \le z \le z_n + l_z \end{matrix} \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

where x_n and z_n are the coordinates of the centre of the n^{th} rooftop and l_x, l_z are the sizes of the grid where the rooftop function is located. The rooftop function is defined by two separable functions, a triangle function in the direction of current flow and a step function in the direction perpendicular to current flow. As shown in figure 1 a rooftop function to model current flow in the z–direction can be expressed as two separable functions, the step function which is is $2l_x$ wide (l_x is the grid size in x direction) and the triangle function which is $2l_z$ wide (l_z is the grid size in z direction). Thus the function overlaps in both directions.

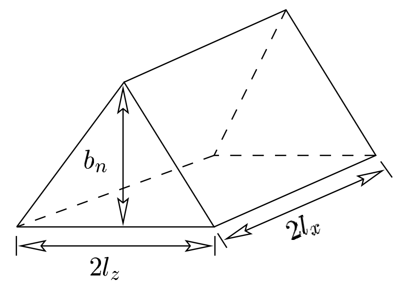

Fig. 1. Z–Directed Rooftop Function

Note that J_{sn} in equations 1, 2 are in the space domain and Fourier transform is required in the Spectral Domain Method. Two dimensional Fourier transform of rooftop functions are given by :

$$J_{xn}(k_x,k_z) = \frac{4}{k_x^2 k_z l_x}(1-\cos(k_x l_x))\sin(k_z l_z)e^{j(k_x x_n + k_z z_n)} \quad (3)$$

$$J_{zn}(k_x,k_z) = \frac{4}{k_x k_z^2 l_z}(1-\cos(k_z l_z))\sin(k_x l_x)e^{j(k_x x_n + k_z z_n)} \quad (4)$$

The sizes of the rooftop function (l_x, l_z) are defined in this contribution as functions of its location (x_n, z_n) allowing sub–gridding so that rapid changes in current distribution are more accurately modelled. The basis philosophy of the sub–gridding is illustrated in figure 2 by taking a re–entrant corner discontinuity as an example. As shown in figure 2 finer rooftops are used where rapid change in current distribution occurs and a course size where only slow changes in current distribution occur.

The rooftop functions define an area of metalisation of the circuit by forming a grid of overlapping rectangular sub–domains as illustrated in figure 3. Each function overlaps in its neighbouring rooftops in both directions. Hence the x and z directed rooftop components are defined on the same grid. It is emphasised that only fine rooftops defined in the interface between course–grid region and sub–grid region overlap into neighbouring regions.

B. Inclusion of A Priori Knowledge

The inclusion of *a priori* knowledge of current distribution and the use of pre–calculated basis functions reduces

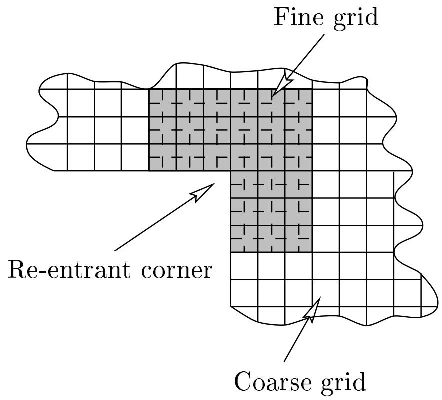

Fig. 2. Illustration of sub–gridding

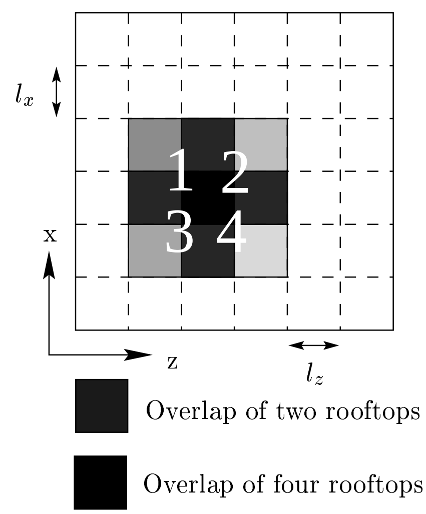

Fig. 3. Illustration of sub–domain basis function gridding

the number of current basis functions required. These can be pre–calculated discontinuity function [6] as a linear combination of the rooftop functions or pre–calculated current wave function [9] with pre–calculated wave constant by the two dimensional version of the technique. Following [5], the current in the circuit is expanded as a linear combination of basis functions,

$$\mathbf{J}(\mathbf{r}) = \sum_{\mathbf{m=1}}^{\mathbf{M}} \mathbf{a_m} \boldsymbol{\psi}_{\mathbf{m}}(\mathbf{r}) \tag{5}$$

where $\psi_m(r)$ is the current distribution of m^{th} basis function. A basis function $\psi_m(r)$ can be the rooftop basis function, the pre–calculated basis function as a linear combination of the rooftop functions and the current wave function with pre–calculated wave constant. It is emphasised that for each basis function the dimensions are not necessary to be identical.

III. Comparison With Other Implementations

In common with this implementation, sub–domain basis functions are used to allow the definition of an arbitrary shaped metalisation of the circuit in [3–6,8,10]. The methods generally use a rectangular gridding system to define the location of the sub–domain function. There are two forms of rectangular grid basis functions implemented, piecewise linear [5,6,8,10] and piecewise sinusoid [3,4] and both are referred to as *rooftop* functions but in this research this term is reserved for the previous piecewise linear definition. The later piecewise sinusoidal sub–domain rooftop functions are commonly used in the space domain Integral Equation Method (IEM), because the Fourier transform is not required. The piecewise linear rooftop functions are more suited to the spectral domain formulation.

As mentioned in section II-A, a rooftop function is defined as two separable functions, a triangle function in the direction to current flow and a step function in the direction perpendicular to current flow (see in figure 1). As illustrated in figure 3, a rooftop function in this contribution is $2l_x$ wide (l_x is the grid size in x direction and x direction is the direction perpendicular to current flow) compared to l_x. Thus the rooftop function overlaps in both directions similar to [5,6] and in contrast to [8,10] in which it overlaps only in the direction of current flow.

Moreover in this implementation the dimensions of the rooftop function are defined as functions of its locations. Finer rooftops are used where rapid change in current distribution occurs and a course size where only slow changes in current distribution occur, as illustrated for re–entrant corner discontinuity in figure 2. The fine grid is next to discontinuity and a coarser grid is defined in the area where the current distribution returns to that of a simple steady state position. This approach is not favoured in all available contributions, for example in [5,6] the grid sizes must be identical to exploit the benefit of using the Fast Fourier transform (FFT).

IV. Numerical Examples

A major difference between the boxed and the open case is that in the former the discrete Fourier transform is employed whereas for the later the continuous transform is required. Therefore the benefits exploited by using the FFT are not available. In this implementations the sub–gridding is employed for the analysis of complex metalisation patterns. The sub–gridding in a sense the rooftop functions is defined as a function of its location. Fine rooftop is used next to the discontinuity. Moreover the pre–calculated current basis functions are used where only slow changes in the current distribution occur. As a test of confidence the microstrip step discontinuity shown in figure 4 is taken an example structure. The planar circuit in figure 4 is completely open on a substrate thickness 1.272mm and relative permitivity 10. The other dimensions are given in figure 4.

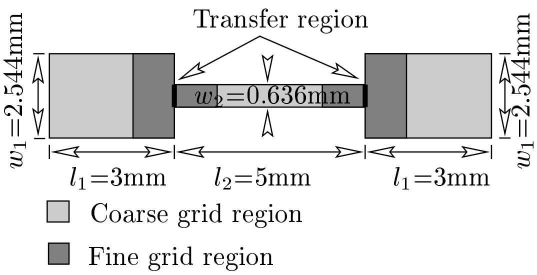

Fig. 4. Microstrip step discontinuity

Three sets of S–parameter results are plotted in figure 5. To illustrate the convergence pattern the circuit in figure 4 is analysed by using fine rooftop functions for entire metalisation pattern and 158 in total (79 z and 79 x components) are required. For the course grid analysis, only three current wave basis functions and 2 transfer current basis functions are used. In the sub–grid analysis, the benefits of using pre–calculated region basis function are exploited an almost same accuracy has been yielded and 35 current basis functions in total are required.

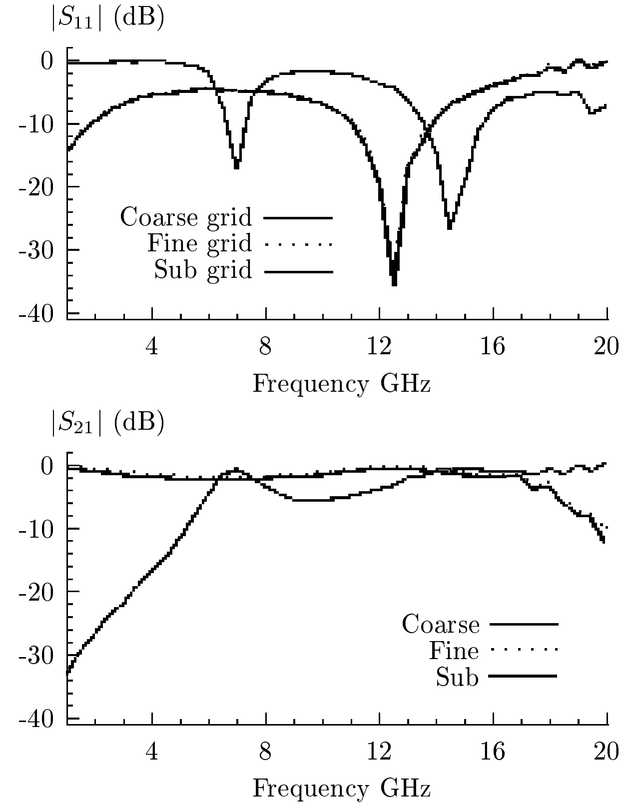

Fig. 5. Plot of S–parameter magnitude for step discontinuity

V. Conclusion

It is shown that realistically complex planar microwave circuits as well as planar antennas can be rigorously analysed by the means of the Spectral Domain Method in combination with the sub–gridding and pre–calculated basis functions.

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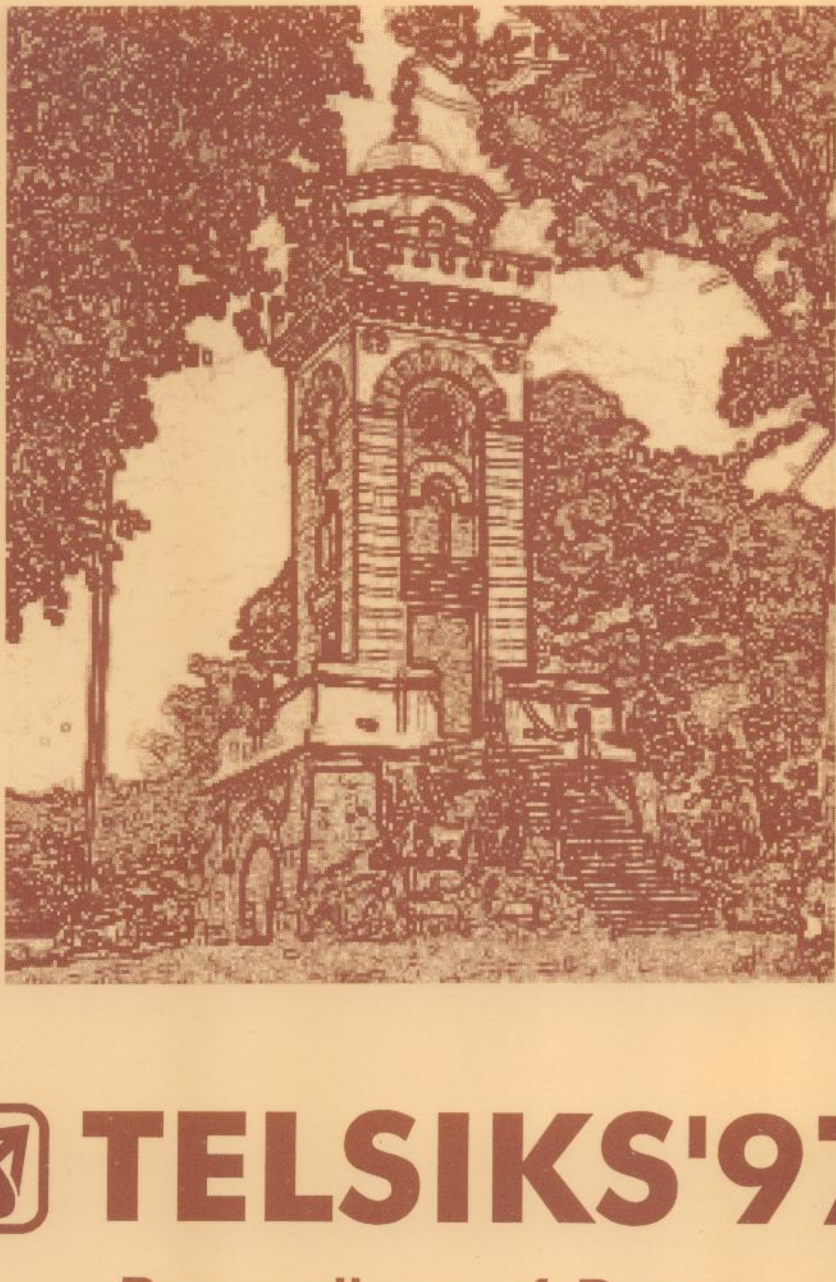

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