

# Adaptive Numerical Integration Technique for the Analysis of Open Planar Circuits and Antennas

Hasan H. Balik and C. J. Railton  
*Student Member IEEE, Member IEEE*  
Room No. 0.14 Queens Building  
University of Bristol  
Centre of Communications Research  
Faculty of Engineering, Bristol, England BS8 1TR  
tel : +44 117 9287738

e-mail: balik@comms-research.bristol.ac.uk, chris@comms-research.bristol.ac.uk

**Abstract:** This paper presents new adaptive integration technique in the spectral domain to speed up the impedance matrix calculation. This includes the adaptive truncation which limits the infinite integration into the finite computer resources and the adaptive integration step which is a function of the Fourier transform variables.

## 1 Introduction

Designers of microwave circuits have come depend heavily on computer-aided-techniques to predict the behaviour of planar microwave circuits and antennas. This is because most of the practical problems can be solved numerically but can not be solved analytically. The fundamental purpose of this paper is to develop an interactive design tool to predict the frequency response of open planar structures.

The Spectral Domain Method (SDM), which is based on solving the coupled integral equations, has been chosen to meet these requirements. The main advantage of SDM is to reduce the coupled integral equation to a simpler set of algebraic equations by taking the Fourier transform. In addition for the planar circuits of interest in this paper, a convenient form of the Green's function exists in the spectral domain.

Analysis of complex planar circuits by the SDM requires the definition of the unknown current distribution on the metal part of the circuit. Therefore the first step of the analysis is to expand the unknown surface current as a set of known basis functions with unknown coefficients. The choice of the basis function is crucial to the efficiency of the technique and special care must be taken to approximate the unknown current distribution as closely as possible, otherwise a large number of basis functions are required for convergence. It has been shown and commonly used that a rooftop function [1, 2] as a current basis function allows the unknown surface current on an irregular shaped metalisation to be defined, but this approach results in a large numbers of basis functions for convergence.

The use of pre-calculated basis functions reduces the number of basis functions required and the impedance matrix to be calculated, but pre-calculated basis functions must be a combination of rooftop functions which must be identical apart from a shift in origin to exploit the benefits of the FFT [3]. Since the FFT is not available for open structures, Balik [4] has recently introduced sub-gridding in the spectral domain and combination of the sub-gridding and the pre-calculated basis function to remedy this deficiency.

Method of Moments [5, chapter5] is applied to find the unknown current coefficients. The Method of Moments requires two dimensional numerical integration over an infinite surface because of open structure. In addition this numerical integration must be repeated at each operating frequency point. The speed of the impedance matrix calculation is a function of the numerical integration, therefore any improvement in efficiency in the numerical integration will result in a significant reduction in the run-time required.

In this paper, an efficient adaptive truncation for the numerical integration to find the impedance matrix elements is presented in order to limit the integration over an infinite surface to finite computer resources by using the features of the current basis function. The present implementation uses the asymptotic form of the Green's function [3] to the calculation for the frequency independent part of the impedance matrix, resulting in a much smaller two dimensional numerical integration to be calculated for each spot frequency. It must be emphasised that the efficient truncation of the integration has already been employed, therefore special care must be taken to determine the integration range of the dyadic part of the impedance matrix.

The accuracy of the numerical integration is also a function of the numerical integration step size. In this contribution these step sizes are defined as functions of the Fourier transform variables. The accuracy has been improved even though fewer steps are used compared to the case of using a uniform step size.

## 2 Adaptive Integration

To find the frequency response of the circuit, equation 1 must be repeated at each frequency point. This requires two dimensional numerical integration and special care must be taken, because the dyadic Green function [6, pages 335-340] has several poles. These poles (in practical cases, only one [2]) correspond to surface waves for an open structure and are located in between  $k_0$  and  $\sqrt{\epsilon_r \mu_r} k_0$ . The poles have no imaginary parts if the dielectric substrate is lossless.

$$\mathbf{Z}_{st} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{w}_t(k_x, k_z) \mathbf{G}(k_x, k_z, d, \omega) \mathbf{J}_s(k_x, k_z) dk_x dk_z \quad (1)$$

where  $\mathbf{G}$  is the dyadic Green's function in the spectral domain,  $\mathbf{J}$  is the Fourier transform of the current basis function and  $\mathbf{w}$  is the weighting function which is identical to the current basis function if the procedure is Galerkin's. It must be noted that **bold** quantities are the Fourier transform of the functions.

There are two possible ways to include such effects described in the literature. The first way is to determine the exact pole location by using one of the numerical root finding techniques such as the Newton-Rhapson procedure, which is well-explained in any numerical analysis

book, then to skip the poles and to include the effect of poles as explained in [7]. The main disadvantage of this technique is to calculate the location of the poles for each spot frequency. A second and more efficient technique is introduced in [8] for the polar coordinate system and therefore enhanced to fit the cartesian coordinate system used in this implementation as illustrated in figure 1.

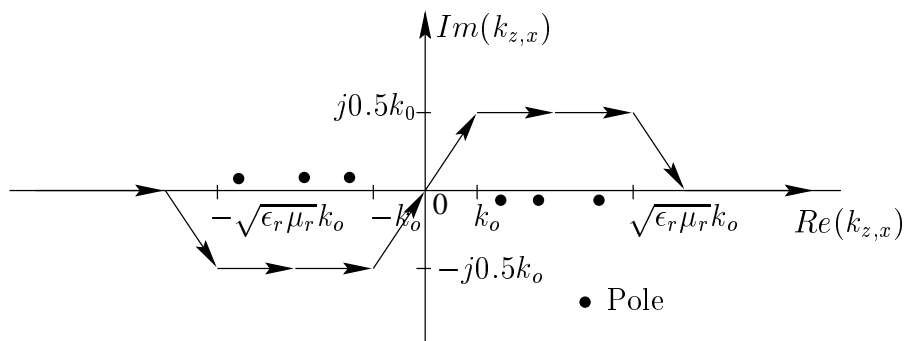


Figure 1: Complex integration path

In a line integration, the result is independent of the chosen path [9, page 409]. The technique used here is based on changing the integration path to a new path on which no poles exist. The poles are located between  $|k_o|$  and  $|\sqrt{\epsilon_r \mu_r} k_0|$  as shown in figure 1. The contour integration path which skip the region where poles are located by shifting integration path 5% of  $k_0$  as shown in figure 1. 5% has been found by the author experience and therefore the effects of poles are included in the analysis.

In the numerical integration procedure, there are two parameters which are most effective. These parameters are given in the following sections and described with their enhancements.

## 2.1 Adaptive Integration Range

To find the unknown current distribution on the complex metalisation of the circuit, the unknown coefficients must be known and hence the Method of Moments (MoM) is commonly used to calculate these coefficients and requires an integration over an infinite surface due to the open structure [2]. A suitable place for termination of the integration must be defined to limit the infinite integration to finite computer resources. Although some truncations were mentioned in [7], the exact position of the truncation was not given. In this contribution the location of the truncation is defined using the feature of the rooftop basis function which is a rooftop function.

A rooftop function is defined as two separable functions, a triangle function in the direction of current flow and a step function in the direction perpendicular to flow [1]. As shown in figure 2 the Fourier transform of the components of the rooftop function become very small after just a few cycles. The integration over a just a few cycles has been found and proved to give accurate results instead of the integration over an infinite surface.

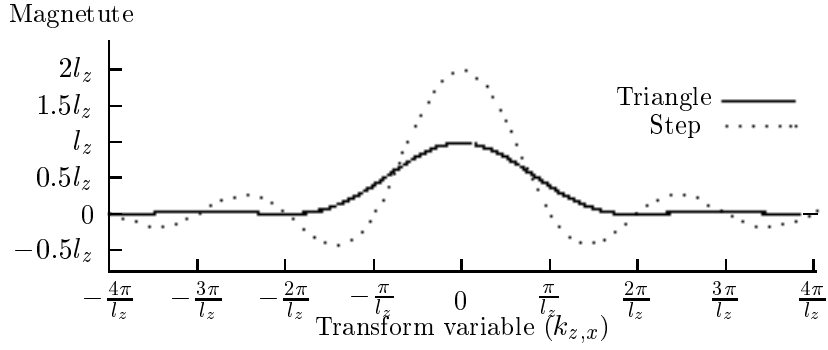


Figure 2: Fourier transform of a rooftop function components

## 2.2 Adaptive Integration Step

The maximum value of each cycle decays exponentially as seen in figure 2, therefore to define an adaptive integration step as a function of the transform variables ( $k_x, k_z$ ) is meaningful. To integrate the impedance matrix elements efficiently the following idea is used. Use a fine step in the large amplitude variation areas and a coarse step in small amplitude variations. The accuracy of this method is comparable to that of using only fine steps.

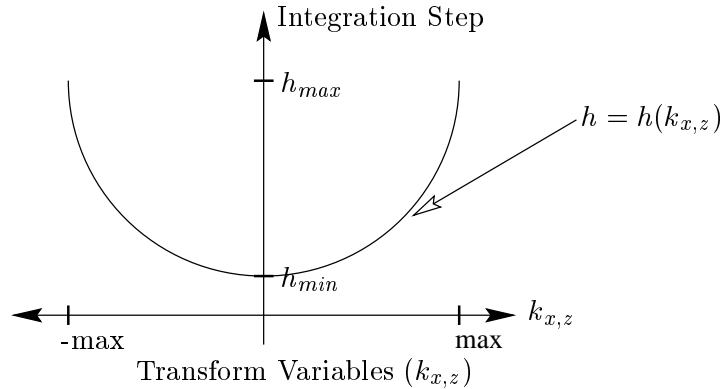


Figure 3: Adaptive integration step

In figure 3,  $max$  is the truncation position which is defined in section 2.1 and the integration step as a function of transform variables is defined as,

$$h(k_{x,z}) = h_{max} e^{\ln\left(\frac{h_{min}}{h_{max}}\right) \frac{|k_{x,z}|}{max}} \quad (2)$$

As seen in equation 2 and figure 3, fine integration steps are used for the small values of the *Fourier* transform variables and coarse steps are used for large values of the *Fourier* transform variables. This allows the user to define  $h_{max}$  and  $h_{min}$ , the integration steps are then automatically calculated.

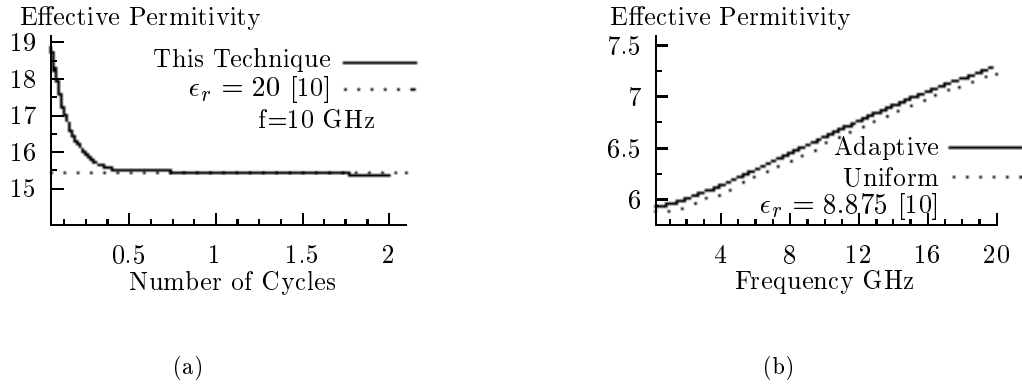


Figure 4: Effective permittivity of open microstrip line

### 3 Numerical Results

To illustrate the convergence pattern, results are presented for the example two dimensional open microstrip structure quoted by Itoh in [10]. In figure 4(a) the effective permittivity versus the number of cycles of the Fourier transform of the rooftop function is plotted. It is evident that relative convergence for this example has been reached when  $N_c \geq 1$  ( $N_c$  is the number of cycles), that is only a small percentage error compared to large values of  $N_c$ . The exact value of  $N_c$  required for convergence is problem dependant but is usually of the order of 1 even more complicated three dimensional structures.

The convergence pattern for adaptive integration step size is shown in figure 4(b). In this case the numbers of integration steps are kept constant. Even though identical numbers of integration step are used, the accuracy is improved by using the adaptive integration technique.

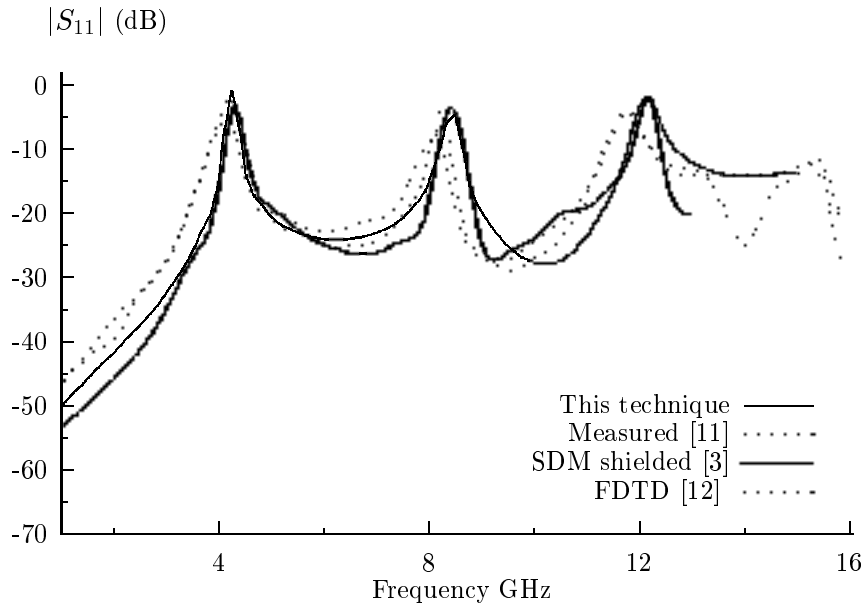


Figure 5: S-parameters  $|S_{21}|$  for edge coupled filter

The present implementation is applied to the edge-coupled filter which is analysed by various techniques. The results for this filter are available from measurements performed by Shibata *et al* [11]. It is also analysed using FDTD [12] and SDM [3] for shielded circuits.

In fig. 3, the problems caused by box modes using the method of [3] have been eliminated. Using the adaptive integration makes the calculation at least 2.5 times faster. The run time on HP workstations is approximately 10 min for the first spot frequency and 5 min for the rest of the each frequency.

## References

- [1] S. A. Meade and C. J. Railton, "Efficient implementation of the spectral domain method including precalculated corner basis functions," *IEEE Transaction on Microwave Theory and Technique*, vol. 42, pp. 1678–1684, September 1994.
- [2] R. W. Jackson, "Full-wave, finite element analysis of irregular microstrip discontinuities," *IEEE Transaction on Microwave Theory and Technique*, vol. 37, pp. 81–89, January 1989.
- [3] C. J. Railton and S. A. Meade, "Fast rigorous analysis of shielded planar filters," *IEEE Transaction on Microwave Theory and Technique*, vol. 40, pp. 978–985, May 1992.
- [4] H. H. Balik and C. J. Railton, "Sub-gridding in the spectral domain method for the analysis of planar circuits and antennas," in *3rd International conference on telecommunications in modern satellite, cable and broadcasting services*, pp. 592–595, October 1997.
- [5] M. N. O. Sadiku, *Numerical Techniques in Electromagnetics*. CRC Press, 1982.
- [6] T. Itoh, *Numerical Techniques for Microwave and Millimeter-Wave Passive Structures*. John Willey and Sons, 1989.
- [7] D. M. Pozar, "Input impedance and mutual coupling of rectangular microstrip antennas," *IEEE Transaction on Antennas and Propagation*, vol. 30, pp. 1191–1196, November 1982.
- [8] E. H. Newman and D. Forrai, "Scattering from a microstrip patch," *IEEE Transaction on Antennas and Propagation*, vol. 35, pp. 245–251, March 1987.
- [9] E. Kreyszing, *Advanced Engineering Mathematics*. Jhon Willy, 5. ed., 1983.
- [10] T. Itoh and R. Mittra, "Spectral-domain approach for calculating the dispersion characteristics of microstrip lines," *IEEE Transaction on Microwave Theory and Technique*, pp. 496–499, July 1973.
- [11] T. Shibata, T. Hayashi, and T. Kimura, "Analysis of microstrip circuits using three-dimensional full-wave electromagnetic field analysis in the time domain," *IEEE Transaction on Microwave Theory and Technique*, vol. 36, pp. 1064–1070, June 1988.

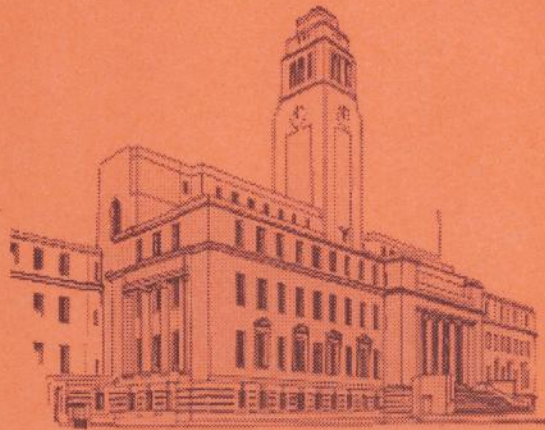
- [12] D. L. Paul, E. M. Daniel, and C. J. Railton, "Fast finite difference time domain method for the analysis of planar microstrip circuits," in *21st European Microwave Conference*, pp. 303–308, September 1991.



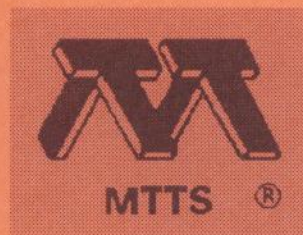
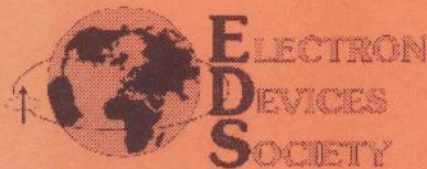
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