# Accurate Analysis of Dielectric Loaded Rectangular Waveguides by 2D-FDTD Method

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***Abstract*- In this paper, fast an efficient numerical analysis method is introduced to analyze dielectric loaded waveguides. 2D-FDTD method is enhanced and improved to make the analysis faster while retaining full-wave technique capabilities. It is believed that introduced technique which is accurate, yet retains the interactive design capabilities of the simpler techniques.**

## I. INTRODUCTION

The problem of electromagnetic wave propagation inside waveguides which is loaded with dielectric has been the subject of many research works over the last years [1-4]. There are varieties of numerical approximation methods employed to solve electromagnetic structures. Some of these techniques solve the problem in the time domain [5, 6] whereas others solve in frequency domain [7, 8]. At recent operating frequencies, full-wave numerical methods which include all fields effects on solution must be employed to get accurate results. Although full-wave numerical technique gives accurate results, it requires more time and computer resources for solutions. The demands of the design engineer require a technique which is accurate, yet retains the interactive design capabilities of the simpler techniques.

The large majority of time-domain simulators, however, is based either on a discretization of Maxwell's equations in differential form (FDTD) or on a discrete spatial network model (TLM). FDTD is an appropriate technique for time-domain analysis of passive microwave and millimeter wave structures. FDTD can be successfully applied to waveguide problems [9, 10]. Discrete time domain models of electromagnetic fields can be obtained by discretizing time domain differential or integral formulations, by discrete spatial network representation of fields.

In this contribution, full-wave 2D-FDTD method has been enhanced and optimized to fulfill interactive design tool capabilities. To show the accuracy, WR284 rectangular waveguide which is loaded with dielectric has been numerically analyzed by introduced 2D-FDTD method. The frequency response of waveguide is obtained using Fast Fourier Transform and zero-padding techniques. It has been shown that the introduced technique is accurate to find mode cutoff frequency. In addition, this paper also shows the effect of relative permittivity variation on cut-off frequency of TE and TM mode electromagnetic waves.

## II. DIELECTRIC RECTANGULAR WAVEGUIDES

Rectangular waveguides are capable of handling very high-power microwave signals, and they are suitable for operations in outdoor environments. The modes supported are referred to as transverse electric (TE) modes and transverse magnetic (TM) modes. The presence of magnetic or electric field components along the propagation direction requires that the wave field components vary with position along the planar phase fronts. TE and TM modes are the only types of electromagnetic waves that can propagate in metallic tube waveguides, which confine the wave energy in both of the transverse directions. Therefore the rectangular waveguide can propagate TM and TE modes, but not TEM waves.

Dielectric filled metallic waveguide has the same property as the air filled one with the only exception that the cutoff frequency for the modes is lower due to the presence of the dielectric. Rectangular waveguide, which is used in the analysis of the electromagnetic wave propagation inside the tube, with internal dimensions a and b is illustrated in Fig. 1 with Cartesian co-ordinate system. It will be assumed that the waveguide has uniform cross section with perfectly conducting walls and there are no singularities in the propagation direction which is z direction, the direction of propagation of the wave. Moreover, inside of the waveguide is loaded with dielectric of which relative permittivity is grater than one.

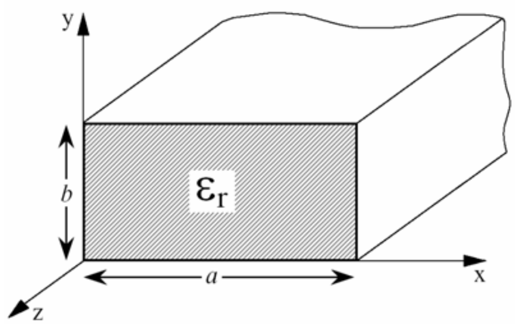


Fig. 1. The cross section of dielectric filled rectangular waveguide.

By expanding Maxwell's equations in a rectangular co-ordinate system and rearranging, it is shown that each of the field components $E_x$, $E_y$, $H_x$ and $H_y$ can be expressed in terms of longitudinally directed components $E_z$ and $H_z$.

Two independent wave equations for $E_z$ and $H_z$ are derived and showed that the analysis can be divided into two basic types or modes of propagating waves. In the first case, there is no magnetic field on the direction of propagation and therefore all fields components can be expresses in term of the electric field components which is in the direction of propagation. This mode of propagation is called a transverse magnetic or TM mode (for which $H_z$=0 and $E_z \neq 0$). In the second case the electric field is entirely transverse but there is an axial component of magnetic field. This mode of propagation is called a transverse electric or TE mode (for which $E_z$=0 and $H_z \neq 0$).

Wave equations for $E_z$ and $H_z$ are shown in (1) and (2), respectively.

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + k_c^2 E_z = 0 \tag{1}$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + k_c^2 H_z = 0 \tag{2}$$

The propagation constant in z direction is given in (3),

$$\gamma_{mn} = \sqrt{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] - \left[\omega^2 \mu \varepsilon\right]} \tag{3}$$

Equation (3) defines an infinite number of modes dependent on the given values of m and n. However, only a finite number will be transmission modes since in this case $\gamma$ must be purely imaginary and so the frequency must be high enough. The mode frequency must be higher than cut-off frequency ($f_c$) so that electromagnetic waves can propagate trough waveguides. If mode frequency less than cutoff frequency, electromagnetic waves decay rapidly in the direction of waveguide axes. In the waveguide there is infinite number of modes every which has different cutoff frequency from others.

The mode cutoff frequency of rectangular waveguide is calculated by (4).

$$f_{c,mn} = \frac{1}{2\pi\sqrt{\mu\varepsilon_0\varepsilon_r}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \tag{4}$$

where m and n are mode degrees respectively.

## III. ENHANCED 2-D FDTD ALGORITHM

FDTD is a direct full-wave numerical solution of Maxwell's time-dependent curl equations. It applies simple, second order accurate central difference approximations for the space and time derivatives of the electric and magnetic fields directly to the respective differential operators of the curl equations.

The original FDTD algorithm proposed by Yee in 1966 is specially designed to solve electromagnetic problems in a rectangular coordinate system and it assumes a lossless, isotropic and non-dispersive medium. With FDTD, it is possible to analyze EM problems directly in time domain for a broad range of frequencies, only in one simulation. Beside the spatial differences in field components, there is also a half-time step difference between electric and magnetic field components, which is called leapfrog computation [11].

E and H fields are assumed interleaved around a cell whose origin is at the location i, j. Every E field is located 1/2 cell width from the origin in the direction of its orientation; every H field is offset 1/2 cell in each direction except that of its orientation.

### A. *Formulation of TM Mode*

In 2-D FDTD analysis, it is assumed that there are no changes in z direction. In other words $\delta/\delta z$ assumed to be zero [12]. In this case TM mode equations for homogeneous and lossless domain ($\sigma$ = 0) can be found as shown in (5), (6) and (7).

$$\frac{\partial H_x}{\partial t} = -\frac{1}{\mu}\frac{\partial E_z}{\partial y} \tag{5}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\frac{\partial E_z}{\partial x} \tag{6}$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) \tag{7}$$

In TM mode, $H_x$, $H_y$ and $E_z$ field components are placed on x-y plane as shown in Fig. 2. The inside walls of waveguide coated with perfectly electrical conductor (PEC). The boundary condition for a perfect electric conductor requires the tangential E field to be zero at the boundary.

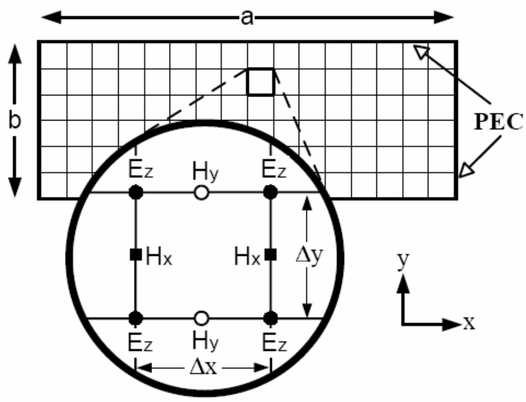


Fig. 2. Position of field components in TM mode.

When (5), (6) and (7) are discretized with central difference approximation, FDTD equations for TM mode are obtained as (8), (9) and (10).

During FDTD simulation of TM mode $H_x$, $H_y$ and $E_z$ field components are calculated iteratively.

$$H_x^{n+1/2}(i,j) = H_x^{n-1/2}(i,j) - \frac{\Delta t}{\mu\Delta y}\left[E_z^n(i,j+1/2) - E_z^n(i,j-1/2)\right] \tag{8}$$

$$H_y^{n+1/2}(i,j) = H_y^{n-1/2}(i,j) + \frac{\Delta t}{\mu\Delta x}\left[E_z^n(i+1/2,j) - E_z^n(i-1/2,j)\right] \tag{9}$$

$$\begin{aligned} E_z^{n+1}(i,j) = E_z^n(i,j) &+ \frac{\Delta t}{\varepsilon(i,j)\Delta x}\left[H_y^{n+1/2}(i+1/2,j) - H_y^{n+1/2}(i-1/2,j)\right] \\ &- \frac{\Delta t}{\varepsilon(i,j)\Delta y}\left[H_x^{n+1/2}(i,j+1/2) - H_x^{n+1/2}(i,j-1/2)\right] \end{aligned} \tag{10}$$

### B. *Formulation of TE Mode*

TE mode equations for homogeneous and lossless domain can be found as shown in (11), (12) and (13).

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\frac{\partial H_z}{\partial y} \tag{11}$$

$$\frac{\partial E_y}{\partial t} = -\frac{1}{\varepsilon}\frac{\partial H_z}{\partial x} \quad (12)$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) \quad (13)$$

In TE mode, $E_x$, $E_y$ and $H_z$ field components are placed on x-y plane as shown in Fig. 3.

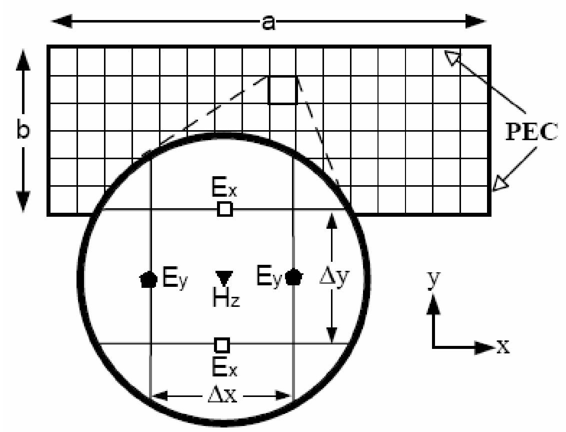


Fig. 3. Position of field components in TE mode.

When (11), (12) and (13) are discretized with central difference approach, FDTD equations for TE mode are obtained as (14), (15) and (16).

Similarly during simulation of TE mode $E_x$, $E_y$ and $H_z$ field components are calculated iteratively.

$$E_x^{n+1/2}(i,j) = E_x^{n-1/2}(i,j) + \frac{\Delta t}{\varepsilon(i,j)\Delta y}\left[H_z^n(i,j+1/2) - H_z^n(i,j-1/2)\right] \quad (14)$$

$$E_y^{n+1/2}(i,j) = E_y^{n-1/2}(i,j) - \frac{\Delta t}{\varepsilon(i,j)\Delta x}\left[H_z^n(i+1/2,j) - H_z^n(i-1/2,j)\right] \quad (15)$$

$$\begin{aligned} H_z^{n+1}(i,j) = H_z^n(i,j) &+ \frac{\Delta t}{\mu\Delta y}\left[E_x^{n+1/2}(i,j+1/2) - E_x^{n+1/2}(i,j-1/2)\right] \\ &- \frac{\Delta t}{\mu\Delta x}\left[E_y^{n+1/2}(i+1/2,j) - E_y^{n+1/2}(i-1/2,j)\right] \end{aligned} \quad (16)$$

## III. NUMERICAL RESULTS

First, max frequency ($f_{max}$) must be determined. Because this process is necessary to specify time step and cell size of FDTD simulation. To avoid numerical dispersion, the ratio of minimum wavelength ($c/f_{max}$) and cell size must be chosen appropriately. This choice is crucial. According to Courant stability criteria time step ($\Delta t$) is determined by cell size.

$$\Delta t \leq \frac{1}{(c/\sqrt{\varepsilon_r}) \cdot \sqrt{(1/\Delta x)^2 + (1/\Delta y)^2}} \quad (17)$$

Therefore waveguide has become divided into $(N_x)\times(N_y)$ number of cells so that $\Delta x$ and $\Delta y$ are cell sizes in x and y axes respectively. As a source, Gauss pulse of which durations are chosen according to maximum frequency used. Inside the FDTD loop, Gauss pulse is applied at one point, and the calculated fields components of observation point are saved. So the time response of dielectric loaded rectangular waveguides is gained. To calculate frequency response of rectangular waveguide, Fourier transform of time response are taken.

In this paper WR284 of which dimensions 72.136mm x 34.036mm have been analyzed by enhance 2D-FDTD. This waveguide are used at S band (2.6GHz-3.95GHz). Dominant mode of WR284 is $TE_{10}$ mode at 2.079GHz.

### *C. TM Mode Analysis Results*

In the simulation of TM mode, the Gauss pulse width is 110ps and the time step used is 3ps. The cell dimensions are $\Delta x$=1.8034 mm and $\Delta y$=1.7018 mm, and total number of cells are 40x20 in x, y directions respectively. The simulation time is 10000$\Delta t$. To increase frequency resolution, 30000 numbers of zeros is added at the end of the data because FDTD simulation is terminated at certain time. Therefore detailed frequency response of the waveguide can be obtained by not extending the simulation so long.

In Fig. 4, the frequency response of the waveguide which is loaded with dielectric whose relative permittivity is 1.0 and 2.32 respectively. Every peak shown in the frequency response at 0-8 GHZ represents mode cutoff frequency.

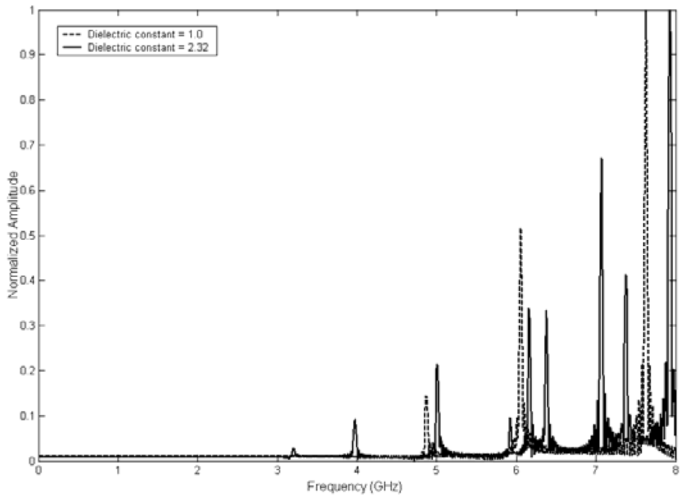


Fig. 4. Frequency response for TM mode.

Table I compares FDTD results and analytical results. In Table I, cut-off frequencies of first 7 modes with least

TABLE I
MODE CUTOFF FREQUENCIES (GHZ) OF DIELECTRIC LOADED RECTANGULAR WAVEGUIDE (TM POLARIZATION)

| Modes | $\varepsilon_r$ =1.0 | | | $\varepsilon_r$ =2.32 | | |
|---|---|---|---|---|---|---|
| | **Exact** | **FDTD** | **Diff. %** | **Exact** | **FDTD** | **Diff. %** |
| $TM_{11}$ | 4.8730 | 4.8690 | 0.082 | 3.1993 | 3.2046 | 0.165 |
| $TM_{21}$ | 6.0596 | 6.0544 | 0.085 | 3.9783 | 3.9748 | 0.087 |
| $TM_{31}$ | 7.6379 | 7.6250 | 0.168 | 5.0145 | 5.0071 | 0.147 |
| $TM_{12}$ | | | | 5.9457 | 5.9214 | 0.408 |
| $TM_{41}$ | | | | 6.1800 | 6.1593 | 0.334 |
| $TM_{22}$ | | | | 6.3986 | 6.3770 | 0.337 |
| $TM_{33}$ | | | | 7.0895 | 7.0685 | 0.296 |

frequencies are given. It is demonstrated that the enhanced FDTD algorithm presented here has in good agreement with analytical results and error is around 0.1%.

*D. TE Mode Analysis Results*

In the simulation of TE mode, the time step, space steps and total simulation time are identical to TM mode simulation parameters. Frequency response of TE modes are given in Fig. 5. Every peak shown on frequency response represents mode cutoff frequency.

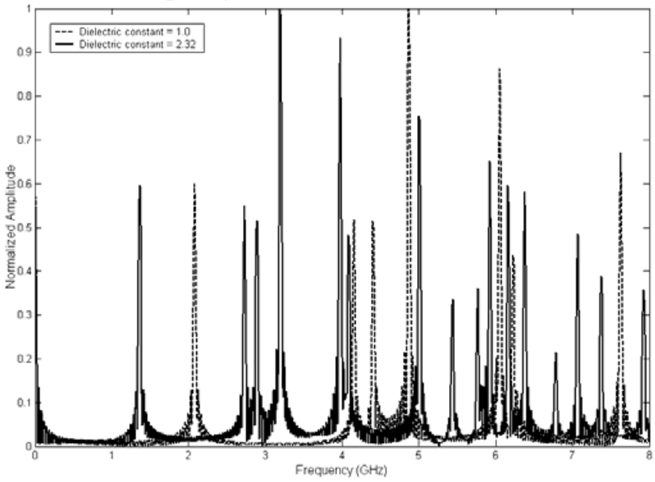


Fig. 5. Time and frequency response for TE mode.

Table II compares FDTD results and analytical results. It is demonstrated that enhanced FDTD algorithm presented here has good agreement and error is around 0.2%.

TABLE II
MODE CUTOFF FREQUENCIES (GHZ) OF DIELECTRIC LOADED RECTANGULAR WAVEGUIDE
(TE POLARIZATION)

| Modes | $\varepsilon_r$ =1.0 | | | $\varepsilon_r$ =2.32 | | |
|---|---|---|---|---|---|---|
| | Exact | FDTD | Diff. % | Exact | FDTD | Diff. % |
| $TE_{10}$ | 2.0794 | 2.0766 | 0.134 | 1.3652 | 1.3669 | 0.124 |
| $TE_{20}$ | 4.1588 | 4.1492 | 0.230 | 2.7304 | 2.7218 | 0.314 |
| $TE_{01}$ | 4.4071 | 4.3992 | 0.179 | 2.8934 | 2.8911 | 0.079 |
| $TE_{11}$ | 4.8730 | 4.8669 | 0.125 | 3.1993 | 3.1976 | 0.053 |
| $TE_{21}$ | 6.0596 | 6.0524 | 0.118 | 3.9783 | 3.9718 | 0.163 |
| $TE_{30}$ | 6.2382 | 6.2218 | 0.262 | 4.0956 | 4.0847 | 0.266 |
| $TE_{31}$ | 7.6379 | 7.6250 | 0.168 | 5.0145 | 5.0040 | 0.209 |

In Fig. 6, there are cutoff frequencies of $TE_{10}$, $TE_{11}$, $TM_{21}$ and $TM_{31}$ modes respectively for different relative permittivity. It is demonstrated that simulation results gained by enhanced 2D-FDTD are in well agreement with analytical results.

## IV. CONCLUSION

In this paper, fast an efficient numerical analysis method was introduced to analyze dielectric loaded waveguides and the variation of mode cutoff frequency versus different relative permittivity has been drawn. It is believed that introduced technique which is accurate, yet retains the interactive design capabilities of the simpler techniques has been developed and its accuracy has been shown by comparing with analytical results.

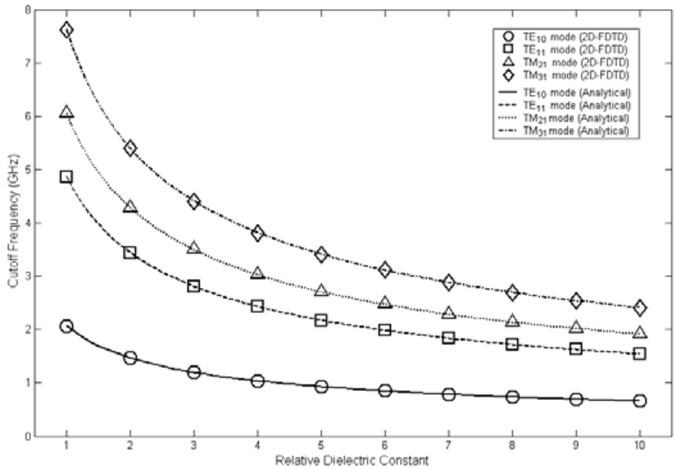


Fig. 6. Variation of mode cutoff frequency versus different relative permittivity.

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