

Numerically Efficient MPIE-MoM Technique for Analysis of Microstrip Structures in Layered Media

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Short Abstract— In this study, full-wave space-domain approach is presented for analysis of some passive microwave circuits in planarly layered media. The Electromagnetic fields are described in terms of a mixed potential integral equation (MPIE) formulation. This formulation is based on the method of moments (MoM) in spatial domain and utilizes closed-Green's functions. We have introduced new microwave filters consists of coupled microstrip open-loop resonators and analyzed to demonstrate the efficiency and accuracy of MPIE-MoM technique.

Keywords-Method of Moments;mixed potential integral equation;microstrip;layered media;Green's functions

I. INTRODUCTION

Passive microwave devices embedded in stratified media have been extensively studied in the literature and numerous planar structures have been designed using integral equation based techniques. Integral equation can be formulated in various ways. Among these, mixed potential integral equation (MPIE) is one of the most successful methods for the analysis of microwave circuits and planar antennas [1-3]. Alternative formulations are the electric field integral equation (EFIE) and magnetic field integral equation (MFIE). For numerical modeling of small-to-medium-sized printed planar geometries used in monolithic integrated microwave and millimeter structures, the method of moments (MoM) [4] is generally accepted to be one of the most efficient and robust methods. The MoM formulation in electromagnetic problems starts with writing the integral equations (MPIE, EFIE or MFIE). Generally, EFIE or MFIE is applied in the spectral-domain, while MPIE is applied in the spatial-domain. It should be noted that EFIE and MFIE use the Green's functions of the electric and magnetic fields as their kernels, whereas the MPIE uses the Green's functions of the scalar and vector potentials, of which singularities are of the order of $1/R$, and, therefore, are less singular [5]. Hence, the use of MPIE in conjunction with the spatial-domain MoM has been preferred for the characterization of planar printed geometries in general. More recently, this formulation was improved by the introduction of suitable closed-form Green's functions in spectral and spatial domains for general media [6]. With the use of closed form Green's functions, calculation of the oscillatory and slow converging Sommerfeld integral is no more necessary. In the derivations, the main goal is to put these closed-form representations in an appropriate form for the solution of MPIE-MoM. MPIE is formulated as the governing equation of

the printed geometries in layered planar media, and the spatial-domain MoM is used to solve for the unknown current densities in the structure.

In this paper, formulation of MPIE-MoM is concisely explained from a theoretical point of view in the next section. Then, scattering parameter analysis and simulation algorithm are introduced. In numerical examples section, the MPIE-MoM formulation is applied to new filter configurations and the results are compared from commercial EM software, em Sonnet. The final section presents the conclusions.

II. FORMULATION OF MPIE-MoM

For the sake of illustration, a typical microstrip structure in layered environment is shown in Figure 1. It is assumed that conductors are lossless and infinitesimally thin and all layers extend to infinity in transverse domain (xy-plane). The thickness and permittivity of each layer are denoted as h_i and ϵ_{ri} , respectively.

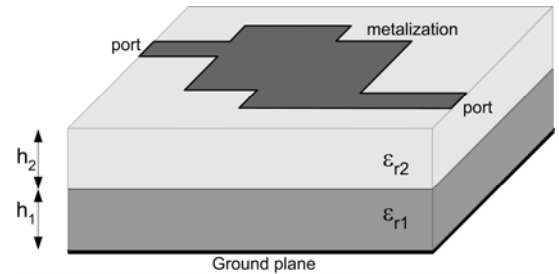


Figure 1. A general microstrip structure in layered media.

In the MPIE-MoM formulation, first the electric field is written in terms of scalar and vector potentials as

$$\mathbf{E} = -j\omega\mathbf{A} - \nabla\phi \quad (1)$$

Then, the vector and scalar potentials are expressed in terms of convolution integrals involving surface density \mathbf{J} and charge density ρ on the metallization as

$$\mathbf{A} = \overline{\mathbf{G}}^A * \mathbf{J} \quad (2)$$

$$\phi = G^q * \rho \quad (3)$$

By employing the continuity equation ($\nabla \cdot \mathbf{J} + j\omega\rho = 0$), the charge density ρ in the scalar potential equation can be written

in terms of surface current density and from (1), the tangential components of electric field on the metallization can obtain as

$$E_x = -j\omega G_{xx}^A * J_x + \frac{1}{j\omega} \frac{\partial}{\partial x} (G^q * \nabla \cdot \mathbf{J}) \quad (4)$$

$$E_y = -j\omega G_{yy}^A * J_y + \frac{1}{j\omega} \frac{\partial}{\partial y} (G^q * \nabla \cdot \mathbf{J}) \quad (5)$$

where * denotes convolution and $G_{xx}^A = G_{yy}^A$. The explicit expression for the Green's functions of the scalar potential in (4) and (5) is:

$$G^q * \nabla \cdot \mathbf{J} = G_x^q * \frac{\partial J_x}{\partial x} + G_y^q * \frac{\partial J_y}{\partial y} \quad (6)$$

Figure 2 shows the block diagram for the solution of MPIE-MoM. In the first step, the unknown current distribution on the metallization is expanded as a set of known basis functions with unknown coefficients or amplitudes as:

$$J_x(x, y) = \sum_m \sum_n I_x^{(m,n)} B_x^{(m,n)}(x, y) \quad (7)$$

$$J_y(x, y) = \sum_m \sum_n I_y^{(m,n)} B_y^{(m,n)}(x, y) \quad (8)$$

where $B_x^{(m,n)}$, $B_y^{(m,n)}$ are the known basis functions with unknown amplitudes $I_x^{(m,n)}$, $I_y^{(m,n)}$, defined at (m,n) -th position on the subdivided horizontal conductor. In this study, the basis functions used to approximate the current density on the metallization are chosen to be rooftop functions, by the use of which the unknown current distribution on metallization can be modeled very accurately.

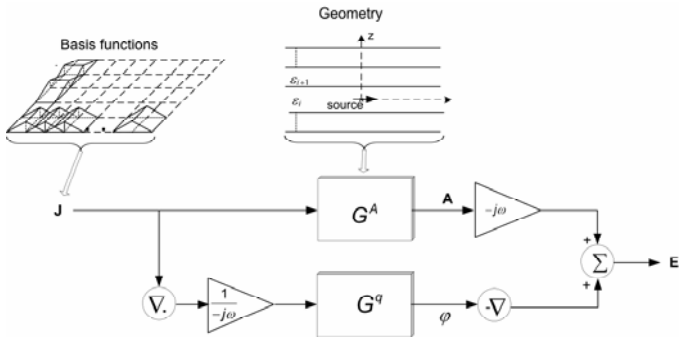


Figure 2. The block diagram for the solution of MPIE-MoM

In the second step, current densities in (7) and (8) are substituted into the electric field expressions of (4) and (5), and boundary conditions are applied. Application of the boundary conditions is performed in the integral sense through the well-known testing procedure of the MoM, where the field expressions are multiplied by testing functions $T_x^{(m',n')}$, $T_y^{(m',n')}$ and integrated on the conductors and set to zero ($\langle T_x^{(m',n')}, E_x \rangle = 0$ and $\langle T_y^{(m',n')}, E_y \rangle = 0$). The resulting matrix equation for the unknown amplitudes of the basis functions has the following form:

$$[Z][I] = [V]$$

$$\begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \begin{bmatrix} I_x \\ I_y \end{bmatrix} = \begin{bmatrix} V_x \\ V_y \end{bmatrix} \quad (9)$$

where $[Z]$ is $N \times N$ impedance matrix and the entries Z_{ij} represent the mutual impedances between the testing and basis functions, $[V]$ is the $N \times 1$ excitation matrix and V 's represent the excitation voltages due to the current source(s), and finally $[I]$ is the $N \times 1$ current coefficient matrix. As an example, a typical matrix term involving both the scalar and vector Green's functions are given in the following form:

$$Z_{xx} = \left\langle T_x^{(m',n')}, G_{xx}^A * B_x^{(m,n)} \right\rangle + \frac{1}{w^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_x^q * \frac{\partial B_x^{(m,n)}}{\partial x} \right] \right\rangle \quad (10)$$

In the above equation, \langle, \rangle denotes inner product and * denotes convolution operator. After forming the matrix entries, two major steps are left to find the unknown coefficients of basis functions: i) evaluation of these matrix entries, ii) solution of the matrix equation for the coefficients of the basis functions. Analytical methods introduced by Alatan et. al [7] are used for the evaluation of these matrix entries. After the evaluation of inner product terms and substituting them into (9), the current densities on the conductors are obtained by solving the matrix equation. Finally, the circuit parameters such as the scattering parameters are extracted from the current distribution.

2.1. Scattering Parameter Analysis

In order to obtain the scattering parameters, a general two port transmission line is used. Having calculated the current densities on the conductor, the current on each port of the transmission line is written as a linear combination of complex exponentials by using the generalized pencil of function (GPOF) method [8] as

$$I(l) \approx \sum_{i=1}^N I_i \cdot e^{(\alpha_i + j\beta_i)l} \quad (11)$$

where l is the distance along the port transmission line, α_i is the attenuation and β_i is the propagation constants of the i^{th} mode of current waves.

2.2. Simulation Algorithm

A simplified flowchart of the algorithm according to the MPIE-MoM solution method described in the previous sections is given in Figure 3. The software starts by reading the layout file that includes the operating frequency, layer information, meshing parameters, and port definitions. According to the meshing parameters, the geometry is subdivided and the number of unknowns is determined. After calculating the coordinates of the basis and test functions, similarities among the inner product terms are tabulated in order to assist the computation in the further steps. Then MoM matrix is filled using the basis functions and Green's functions. The resulting

linear system is solved for the unknown basis amplitudes. Finally, circuit parameters are saved.

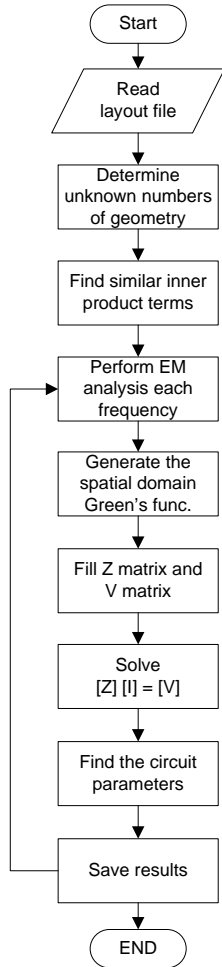


Figure 3. Simple flowchart of the algorithm

III. NUMERICAL EXAMPLES

In this section, the MPIE-MoM formulation is applied to new filter configuration consists of coupled microstrip open-loop resonators. For the examples, general microstrip geometry in a layered media is assumed where all layers and the ground plane extend to infinity in the horizontal plane, and the conductors are lossless and thin. The S-parameters provided here are normalized with respect to 50-Ω reference impedance. The method used in this work is compared with the results obtained from the well-known commercially available full-wave EM Simulator Sonnet.

First example is the dual-mode linear phase filter as shown in Figure 4. The filter consists of a set of microstrip coupled open-loop resonators with a spacing of 1.5mm and an open-gap of 0.5mm on a substrate with a thickness of 1.27mm and $\epsilon_r = 10.2$. The size of open-loop resonator is 6.5mm, the width of coupled open-loop arms is 1mm and the length of the feed line is 3mm. The geometry was analyzed by using the MPIE-MoM technique, over a frequency range of 2.0 GHz to 3.0 GHz.

Figure 5 is comparison of the S-parameters for the dual mode linear phase filter.

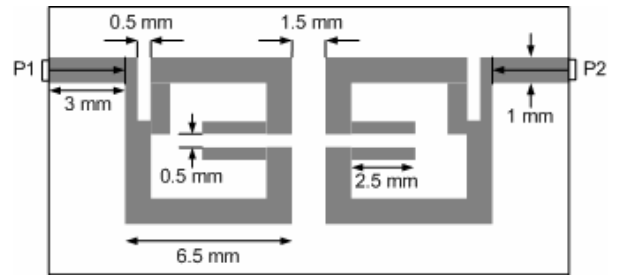


Figure 4. Geometry of the dual-mode linear phase filter.

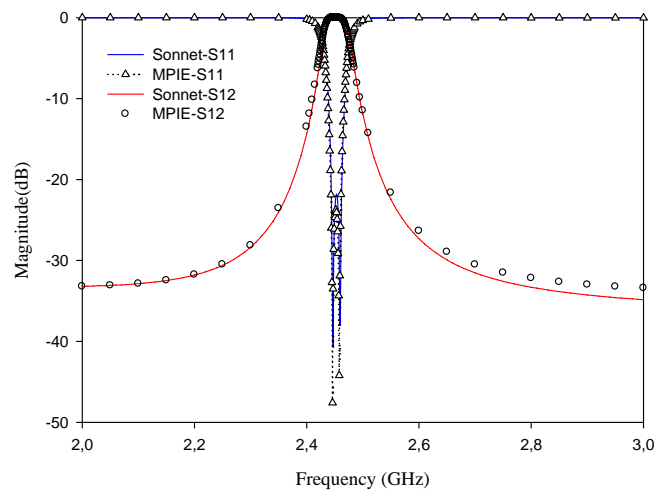


Figure 5. Magnitudes of S_{11} and S_{12} of the dual-mode linear phase filter shown in Figure 4.

Second example is the dual-mode elliptic filter as shown in Figure 6. As shown in Figure 6, the difference from dual-mode linear phase filter is diagonal placement of resonator feed lines. For the same frequency range, analysis results obtained using MPIE-MoM technique is represented in Figure 7.

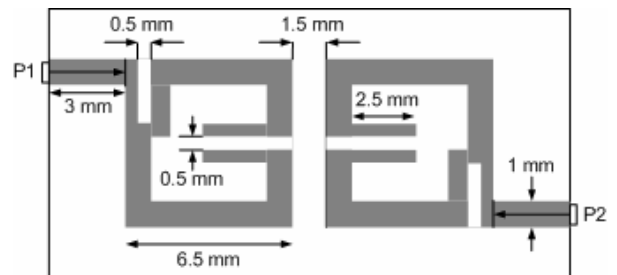


Figure 6. Geometry of dual-mode elliptic filter.

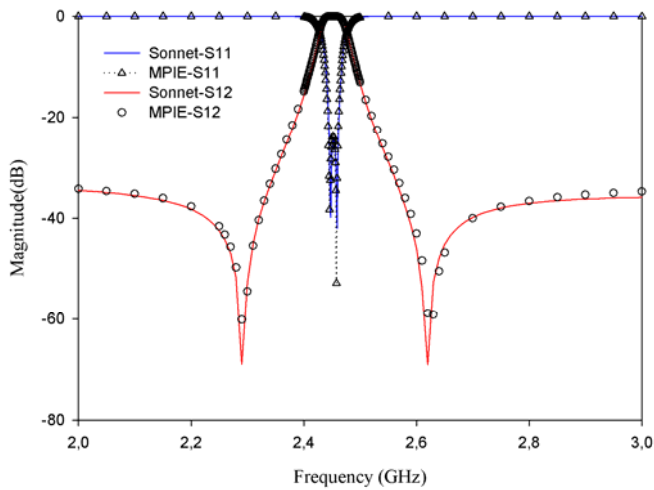


Figure 7. Magnitudes of S_{11} and S_{12} of the dual-mode elliptic filter shown in Figure 4.

When seeing frequency response of filters shown in Figure 5 and 7, both filters represent band-pass filter characteristics. The two types of filters have been constructed from same type of resonators by exchanging feed lines as cross and diagonally.

IV. CONCLUSION

This paper presents a numerically efficient MPIE-MoM solution for analysis of microstrip structures in a layered media. New microwave filters consists of coupled microstrip open-loop resonators are analyzed to demonstrate the efficiency and accuracy of MPIE-MoM technique. The results obtained are in

good agreement with the results obtained from well-known EM software SONNET. Analyzed new designed filters demonstrate linear phase and elliptic frequency characteristics with having narrow band and high selectivity features. Because of these features, these filters were found to be suitable for sensitive microwave circuits needed in mobile communication systems.

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