Fast Rigorous Analysis of Rectangular Waveguides by Optimized 2D-TLM

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Abstract. In this paper, The optimized 2D-TLM as been introduced and applied to rectangular waveguides which is widely used. Results obtained by using optimized 2D-TLM were compared with analytic results and shown to be accurate.

1 Introduction

Rectangular waveguide is one of the earliest type of the transmission lines and still commonly used in many current applications. A lot of components such as isolators, detectors, attenuators, couplers and slotted lines are available to use for various standard waveguide bands between 1 GHz to above 220 GHz [1]. At the operating frequencies where these waveguides commonly used, the assumptions which are valid only low frequencies can not be applied to gain accurate results. Therefore full-wave analysis techniques must be required. Some of these full-wave numerical techniques solve the problem in time domain [2-4] whereas others solve in frequency domain [4-7]. Although full-wave numerical technique gives accurate results, it requires more time and computer resources for solutions. The demands of the design engineer require a technique which is accurate, yet retains the interactive design capabilities of the simpler techniques.

In this contribution, time and frequency domain analysis of rectangular waveguide has been accurately analyzed by enhanced 2-D TLM method and shown to be accurate to find mode cut-off frequency.

2 Review to Rectangular Waveguides

The rectangular waveguide shown in Fig. 1 supports both TM and TE modes; therefore it is not possible to define unique voltage by only using TEM waves at the recent operating frequencies. The waves cannot propagate trough the rectangular waveguide if the operating frequency is below then some certain frequency. This frequency is called cut-off frequency. The mode frequency must be higher then this cut-off frequency. If mode frequency is lower then cut-off frequency, propagating waves decay rapidly in the direction of waveguides axes. When the operating frequency is higher then cut-off frequency, waves have two modes. These are TE and TM modes respectively. The cut-off frequency has been only determined by geometry of the wave guides.

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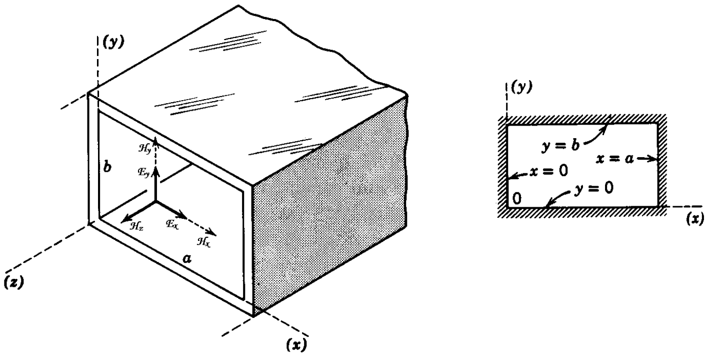

Fig. 1. Rectangular Waveguides

Mode cut-off frequency can be analytically calculated by;

$$f_{c,mn} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \tag{1}$$

where m and n are mode degrees respectively.

3 Optimized 2D-TLM Method

TLM was first introduced by P. N. John in 1970. This technique is based on the field theory – the circuit theory similarities. Transmission line modeling divides the structure into unit cells and structure model is carried out by solving each cell

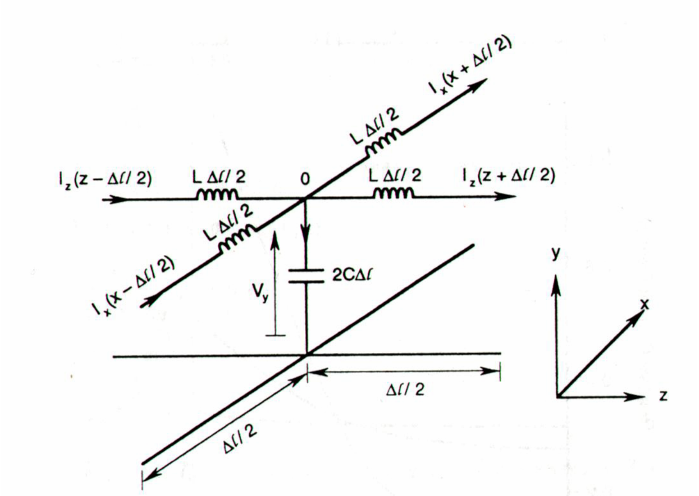

Fig. 2. Two Dimensional TLM Cell

separately. Current and voltage are set to be independent variables. The correlation between input and output voltage is found by applying Kirchoff current and voltage laws onto Fig. 2 which is circuit model of the cell analyzed.

TLM method like FDTD is interested in Maxwell equations. Given microwave structure which is rectangular waveguide in this application has been divided into cells. Each cell has been traded as electrical circuit and therefore electrical circuit solution has been applied onto the every cell repeatedly. The main advantage of TLM against MoM or SDM, TDM does not require any pre-calculation. As a result of this advantage, any optimization can be applied to any microwave circuits without refinements. Another reason to choose TLM for this contribution is that TLM method is very easy to adapt on the computer programming.

2D-TLM Equation is given by;

$$\frac{\partial^2\Phi}{\partial u^2}+\frac{\partial^2\Phi}{\partial v^2}=\mu\varepsilon\frac{\partial^2\Phi}{\partial t^2} \tag{2}$$

For 2D applications $\frac{\partial}{\partial y}=0$ and E_x=E_y=H_y=0. Therefore;

$$\frac{\partial H_x}{\partial z}-\frac{\partial H_z}{\partial x}=+\varepsilon\frac{\partial E_y}{\partial t} \tag{3}$$

$$\frac{\partial^2 E_y}{\partial x^2}+\frac{\partial^2 E_y}{\partial z^2}=\mu\varepsilon\frac{\partial^2 E_y}{\partial t^2} \tag{4}$$

Both equation (3) and equation (4) are very similar. If above equations are rewritten as voltage and current;

$$E_y\equiv V_y\ ,\quad H_z\equiv I_x\ ,\quad H_x\equiv -I_z\ ,\quad \mu=L\ ,\quad \varepsilon=2C \tag{5}$$

$$\mu_r=\varepsilon_r=1 \text{ and } 1/\sqrt{LC}=1/\sqrt{\mu_0\varepsilon_0}=c \tag{6}$$

can be easily found. c is free space light speed in Equation (6).

4 Computer Simulation and Numerical Results

First analyzed mode and then maximum frequency of the interest must be determined. Because this process is necessary to specify time step and cell size of TLM simulation. To avoid numerical dispersion, the ratio of minimum wavelength and cell must be chosen carefully. Rectangular waveguide analyzed by TLM has divided into N_xxN_y number of cells so that Δx and Δy are cell sizes in x –y axes respectively. As a source, Gauss pulse of which durations have been chosen according to maximum operating frequency used. Gauss pulse is applied at one point, and the calculated field's components of observation points are saved. The frequency response of rectangular waveguides has been derived from the time response.

4.1 TM Mode Analysis Results

The analyzed rectangular waveguide's dimensions are given 90mm in width and 45mm in height respectively. Chosen parameters used throughout the computer simulation of TM Mode by optimized TLM technique are given in Table 1

Table 1. 2D-TLM Parameters for TM Modes

f_{max} (maximum frequency)	10 GHz
Δx (cell size in x-axes)	1.125mm
Δy (cell size in y-axes)	1.125mm
N_x (number of cell in x-axes)	80
N_y (number of cell in y-axes)	40
Δt (time step)	2.76 pico second
T (simulation duration)	10000Δt
Δf (frequency resolution)	36.23 MHz

Table 2 compares optimized results and analytical results. It is demonstrated that optimized TLM algorithm presented here has good agreement with analytical results and error is less then 0.2%. Time and frequency response of TM modes are given in Figure 3.

Table 2. 2D-TLM Simulation Results and Analytic Result TM Modes Frequencies

	Analytical Results (GHz)	**TLM Results (GHz)**	**Error**
TM_{11}	**3.7268**	**3,6978**	**0,0290**
TM_{21}	**4.7140**	**4,6994**	**0,0146**
TM_{31}	**6.0093**	**5,9869**	**0,0224**
TM_{12}	**6.8718**	**6,8703**	**0,0015**
TM_{22} TM_{41}	**7.4536**	**7,4155**	**0,0381**
TM_{32}	**8.3333**	**8,3150**	**0,0183**
TM_{51}	**8.9753**	**8,9279**	**0,0474**
TM_{42}	**9.4281**	**9,4059**	**0,0222**

4.2 TE Mode Analysis Results

Chosen parameters used throughout the computer simulation of TE Mode by optimized TLM technique are given in Table 3.

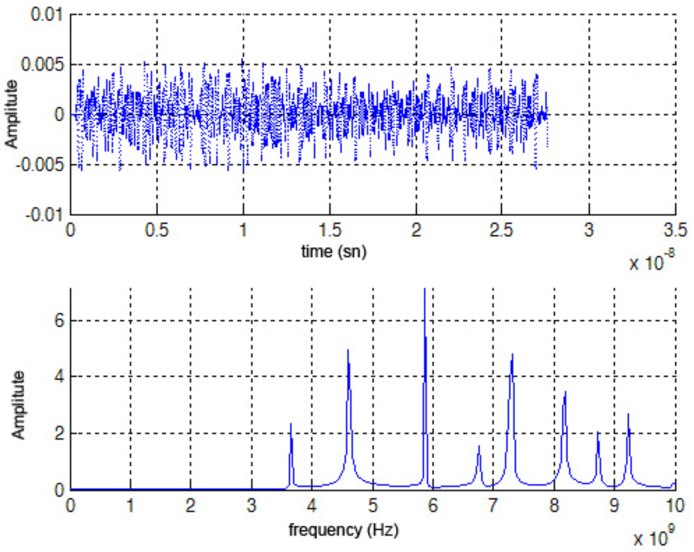

Fig. 3. TM Mode Time and Frequencies Response

Table 3. 2D-TLM Parameters for TE Modes

f_{max} (maximum frequency)	10 GHz
Δx (cell size in x-axes)	2.25mm
Δy (cell size in y-axes)	2.25mm
N_x (number of cell in x-axes)	40
N_y (number of cell in y-axes)	20
Δt (time step)	5.46 pico second
T (simulation duration)	$10000\Delta t$
Δf (frequency resolution)	18.31 MHz

Table 4. 2D-TLM Simulation Results and Analytic Result TE Modes Frequencies

	Analytical Results (GHz)	TLM Results (GHz)	Error
TE_{10}	1.6667	1.6476	0,0191
TE_{01} ve TE_{20}	3.3333	3.2962	0,0371
TE_{11}	3.7268	3.6980	0,0288
TE_{21}	4.7140	4.9001	-0,1861
TE_{30}	5.0000	4.9866	0,0134
TE_{31}	6.0093	5.9670	0,0423
TE_{40} ve TE_{02}	6.6667	6.5540	0,1127
TE_{12}	6.8718	6.8130	0,0588
TE_{41} ve TE_{22}	7.4536	7.3594	0,0942
TE_{50} ve TE_{32}	8.3333	8.2039	0,1294
TE_{51}	8.9753	8.8240	0,1513
TE_{42}	9.4281	9.4002	0,0279

Table 4 compares optimized results and analytical results. It is demonstrated that optimized TLM algorithm presented here has a good agreement with analytical results and error is less then 0.2%. Time and frequency response of TE modes are given in Figure 4.

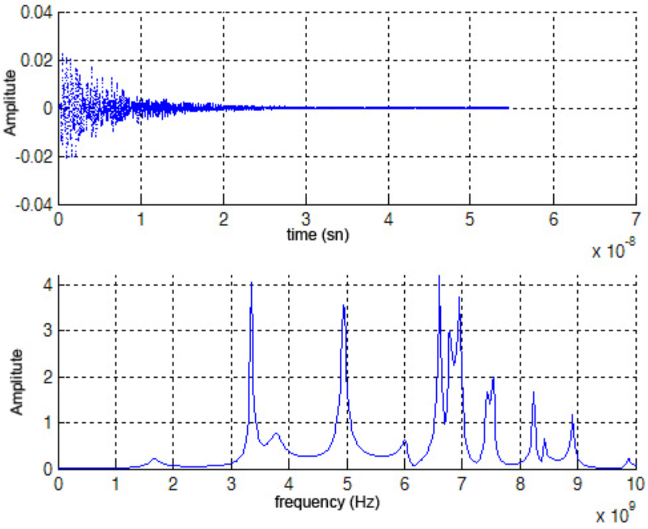

Fig. 4. TE Mode Time and Frequencies Response

5 Conclusion

In this paper, optimized 2D-TLM method has been introduced to analyze widely used rectangular waveguides. It is found and demonstrated in this contribution that the results are in very good agreements to analytical results.

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