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FAST AND ACCURATE ANALYSIS OF DUAL-MODE FILTER CONFIGURATIONS WITH MPIE-MOM TECHNIQUE

Zulfu GENC[1] and Hasan H. BALIK[2*]
[1]Dept. of Informatics, University of Firat, Elazig, TURKEY
zulfugenc@hotmail.com
[2]Dept. of Electronics and Communication Engineering, Beykent University, Istanbul, TURKEY, hasanbalik@gmail.com

[*] Corresponding Author

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Abstract

A full-wave space-domain mixed potential integral equation (MPIE) approach, discredited with the method of moments (MoM) and used in spatial-domain closed-form Green's functions, is presented for the numerically efficient analysis of some passive microwave structures in planary layered medium. The spatial-domain Green's functions for the vector and scalar potentials, represented by the Sommerfeld integrals, are approximated by closed-form expressions and used in the solution of the MPIE by the MoM. By using the closed-form Green's functions into the MoM, the computational efficiency of the MPIE-MoM is significantly improved. The main advantage of the MPIE-MoM technique allows a large variety of printed circuit structures to be characterized. The accuracy and the efficiency of the technique is demonstrated at the example of dual-mode filter configurations consist of coupled microstrip open-loop resonators, and compared the results with those obtained from commercial EM software such as EM Sonnet.

Keywords: Numerical analysis; mixed potential integral equation; method of moments; Green's function; dual-mode microwave filter

1. INTRODUCTION

The study of passive microwave devices embedded in stratified media is crucial in current microwave engineering [1,2] and, in particular, numerous planar devices have been extensively studied with the use of various integral equations and other related techniques [3-5]. Integral equation analysis of planar structures in layered media are formulated as electric field integral equation (EFIE) or mixed potential integral equation (MPIE) with related

Green's functions [6]. Generally, EFIE calculations take place in the spectral-domain while MPIE is applied in the spatial-domain [7]. In addition, the MPIE formulation uses scalar and vector potentials and benefits from the fact that the singularities of both potentials are of the order of 1/R and, therefore, less singular than spatial-domain EFIE formulations [8]. Spatial-domain method of moments [9] (MoM) is one of the most efficient and robust technique for solution of MPIE for printed circuits in planary layered media. The MPIE approach is used in spatial domain closed-form Green's functions and discredited with the MoM. More recently, introduction and fast approximation of the suitable closed-form Green's functions have improved the efficiency of MPIE-MoM [10,11]. Calculation of the oscillatory and slow converging Sommerfeld integral is no more necessary with the use of closed-form Green's functions. In the derivations, main goal is to put these closed-form representations in an appropriate form for the solution of MPIE-MoM.

In this paper, full-wave space domain MPIE-MoM technique is described for the efficient and robust analysis of dual-mode filter structures. MPIE is formulated as the governing equation and the spatial-domain MoM is used to solve for the current densities in the structures.

2. FORMULATION OF MPIE-MOM

Consider a general microstrip structure in layered media, as shown in Figure 1. The microstrip structures are assumed to be of infinitesimal conductor thickness and lossless. The media is assumed to be laterally infinite in transverse domain (xy-plane). The thickness and permittivity of each layer are denoted as hi, and εri, respectively. The unknown current densities on the conductors can be obtained from the MoM solution of pertinent MPIE.

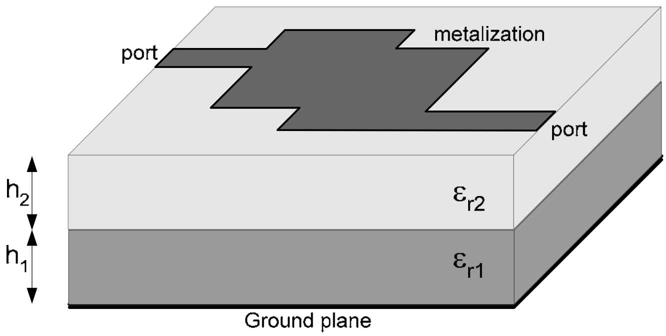


Fig. 1. A general microstrip structure in layered media

The mixed potential integral equation (MPIE) is written in terms of the scalar and the vector potentials as:

$$\boldsymbol{E} = -jw\boldsymbol{A} - \nabla \phi \tag{1}$$

Then, the vector and scalar potentials expressed in terms of convolution integrals involving surface density J and charge density ρ on the metallization in (2) and (3).

$$\boldsymbol{A} = \overline{\boldsymbol{G}}^{A} * \boldsymbol{J} \tag{2}$$

$$\phi = G^{q} * \rho \tag{3}$$

Where GA is the dyadic Green's function of vector potential, Gq is the Green's function of scalar potential. By employing the continuity equation as in (4), the charge density ρ in the scalar potential equation can be written in terms of surface current density and from (1), the tangential components of electric field on the metallization can be obtain as in (5) and (6).

$$\nabla.\boldsymbol{J} + jw\rho = 0 \tag{4}$$

$$E_x = -jwG_{xx}^{A} * J_x + \frac{1}{jw}\frac{\partial}{\partial x}(G^{q} * \nabla.\boldsymbol{J}) \tag{5}$$

$$E_y = -jwG_{yy}^{A} * J_y + \frac{1}{jw}\frac{\partial}{\partial y}(G^{q} * \nabla.\boldsymbol{J}) \tag{6}$$

Where $G_{xx}^{A} = G_{yy}^{A}$. The explicit expression for the Green's functions of scalar potential in (5) and (6) is:

$$G^{q} * \nabla.\boldsymbol{J} = G_x^{q} * \frac{\partial J_x}{\partial x} + G_y^{q} * \frac{\partial J_y}{\partial y} \tag{7}$$

Where $G_x^{q} = G_y^{q}$ is the Green's function of scalar potential for a horizontal electric dipole. Figure 2 shows the block diagram for the solution of MPIE-MoM.

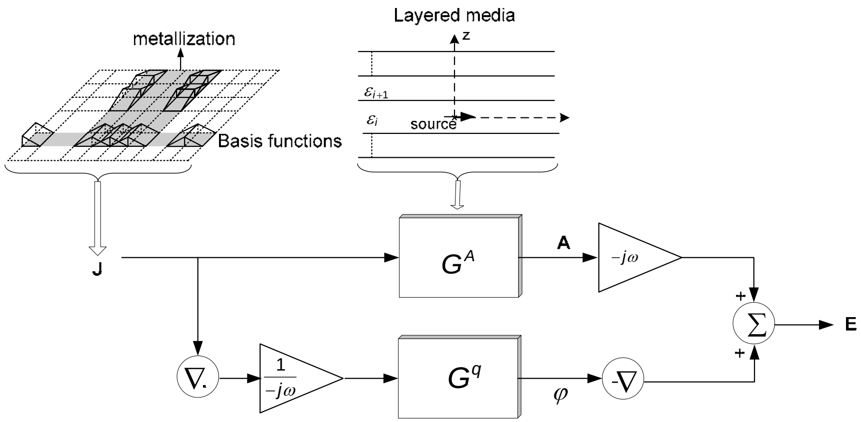


Fig. 2. The block diagram for the solution of MPIE-MoM

As procedure of the MoM, in the solution for the surface current density J on the metallization, J is expanded as a set of known basis functions with unknown coefficients or amplitudes as given as;

$$J_x(x,y) = \sum_m \sum_n I_x^{(m,n)} B_x^{(m,n)}(x,y) \tag{8}$$

$$J_y(x,y) = \sum_m \sum_n I_y^{(m,n)} B_y^{(m,n)}(x,y) \tag{9}$$

$B_x^{(m,n)}, B_y^{(m,n)}$ are the known basis functions $I_x^{(m,n)}, I_y^{(m,n)}$ are unknown coefficients and (m,n)-th is position on the subdivided horizontal conductor. In this study, the basis functions used to approximate the current density on the metallization are chosen to be rooftop functions, by the use of which the unknown current distribution on metallization can be modelled very accurately. The discretization of the geometry and basis functions is shown in Figure 3. Cell is explained as elemental area of circuit metallization. Figure 3 shows each elemental rooftop basis function covers two cells and sources are modelled using a half rooftop. Since the rooftops overlap, the sum of the two rooftops covers three cells. The choices of basis functions are crucial to obtain an accurate and efficient solution of MPIE-MoM [12].

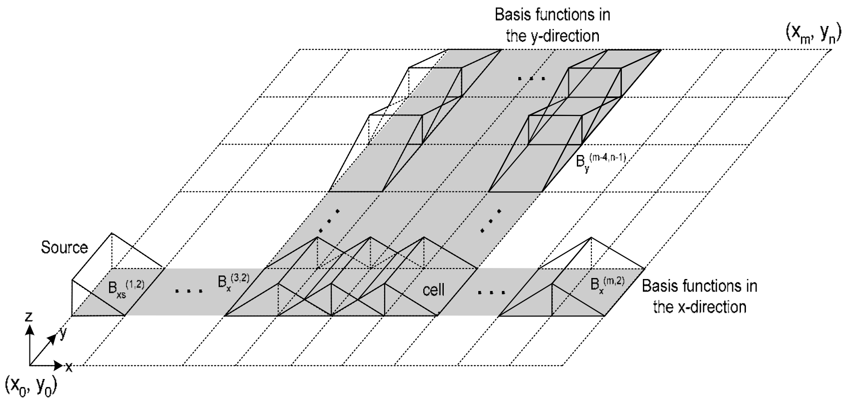


Fig.3. The discretization of the geometry and basis functions.

After substituting the expanded current densities into the electric field expressions of (5) and (6), the boundary conditions are applied. Application of the boundary conditions, which are the tangential electrical field components is identically zero in the metallization of the circuit, are performed in the integral sense through the well-known testing procedure of the MoM, where the field expressions are multiplied by testing functions $T_x^{(m',n')}, T_y^{(m',n')}$ and integrated on the conductors and set to zero. Note that testing functions chosen to be rooftop like basis functions.

$$E_i = -jwG_{ii}^A * \left\{ \sum_m \sum_n I_i^{(m,n)} B_i^{(m,n)} + \sum_n I_{iS}^{(n)} B_{iS}^{(n)} \right\}$$
$$+ \frac{1}{jw} \frac{\partial}{\partial i} \left(G_x^q * \frac{\partial}{\partial x} \left\{ \sum_m \sum_n I_x^{(m,n)} B_x^{(m,n)} + \sum_n I_{xS}^{(n)} B_{xS}^{(n)} \right\} \right) \quad (10)$$
$$+ \frac{1}{jw} \frac{\partial}{\partial i} \left(G_y^q * \frac{\partial}{\partial y} \left\{ \sum_m \sum_n I_y^{(m,n)} B_y^{(m,n)} + \sum_n I_{yS}^{(m)} B_{yS}^{(m)} \right\} \right)$$

Testing of Ei (i=x,y), applying boundary conditions in on the horizontal conductor is denoted as follows:

$$\left\langle T_x^{(m',n')}, E_x \right\rangle = 0$$
$$\left\langle T_y^{(m',n')}, E_y \right\rangle = 0 \quad (11)$$

For example, after multiplying Ex with related testing function and arranging the terms, the following equation is obtained:

$$\sum_m \sum_n I_x^{(m,n)} \left\{ \left\langle T_x^{(m',n')}, G_{xx}^A * B_x^{(m,n)} \right\rangle + \frac{1}{w^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x}\left[G_x^q * \frac{\partial}{\partial x} B_x^{(m,n)} \right.\right.\right.$$

$$+ \frac{1}{w^2} \sum_m \sum_n I_y^{(m,n)} \left\{ \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x}\left[G_y^q * \frac{\partial}{\partial y} B_y^{(m,n)} \right]\right\rangle \right\}$$

$$+ \sum I_{xS}^{(n)} \left\{ \left\langle T_x^{(m',n')}, G_{xx}^A * B_{xS}^{(n)} \right\rangle + \frac{1}{w^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x}\left[G_x^q * \frac{\partial}{\partial x} B_{xS}^{(n)} \right]\right\rangle \right\} \quad (12)$$

$$+ \frac{1}{w^2} \sum_m I_{yS}^{(m)} \left\{ \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x}\left[G_y^q * \frac{\partial}{\partial y} B_{yS}^{(m)} \right]\right\rangle \right\} = 0$$

In this expression, the term with unknown coefficient $I_x^{(m,n)}$ is elements of the Z_{xx} sub matrix. Similarly term with coefficient $I_y^{(m,n)}$ is entry of the Z_{xy} sub matrix. The terms with coefficients $I_{xS}^{(n)}$ and $I_{yS}^{(m)}$ constitute the elements of the excitation matrix V_x. Note that < , > and * denote inner product and convolution integral, respectively, and they are defined as follows:

$$\left\langle f(x,y), g(x,y) \right\rangle = \iint dxdy\ f^*(x,y) g(x,y) \quad (13)$$

$$f(x,y) * g(x,y) = \iint dx'dy' f(x-x', y-y').g(x',y') \quad (14)$$

As a result of all these steps, two linear equations for the problem are obtained and have the following form:

$$\begin{aligned} Z_{xx} I_x + Z_{xy} I_y &= V_x \\ Z_{yx} I_x + Z_{yy} I_y &= V_y \end{aligned} \quad (15)$$

The resulting matrix equation for the unknown amplitudes of the basis functions has the following form:

$$\underbrace{\begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix}}_{[Z]} \underbrace{\begin{bmatrix} I_x \\ I_y \end{bmatrix}}_{[I]} = \underbrace{\begin{bmatrix} V_x \\ V_y \end{bmatrix}}_{[V]} \quad (16)$$

As an example, a typical matrix term involving both the scalar and the vector Green's functions are given as

$$Z_{xx} = \left\langle T_x^{(m',n')}, G_{xx}^A * B_x^{(m,n)} \right\rangle + \frac{1}{w^2} \left\langle T_x^{(m',n')}, \frac{\partial}{\partial x} \left[G_x^q * \frac{\partial B_x^{(m,n)}}{\partial x} \right] \right\rangle \qquad (17)$$

After forming the matrix entries, two major steps are left to find the unknown coefficients of basis functions: i) evaluation of these matrix entries, ii) solution of the matrix equation for the coefficients of the basis functions. Analytical methods introduced by Alatan et al. [13,14] are used for the evaluation of these matrix entries. After the evaluation of inner product terms and substituting them into (16), the current densities on the conductors are obtained by solving the matrix equation. Well known, computationally expensive LU decomposition algorithm is used for the solution of resulting matrix equation. Finally, the circuit parameters such as the scattering parameters are extracted from the current distribution.

3 GREEN'S FUNCTIONS IN MPIE FORMULATIONS FOR PLANARLY LAYERED MEDIA

Green's functions play an important role in the integral equations for electromagnetic problems. In the application of the spatial-domain MoM to the solution of MPIE, one needs to calculate the vector and scalar potentials Green's functions in the spatial-domain. The Green's functions in the spatial domain are obtained from their frequency domain counterparts with the use of an integral transformation called the Hankel transform or the Sommerfeld integral [10]. This transformation is given in the following equation:

$$G = \frac{1}{4\pi} \int_{SIP} dk_\rho k_\rho H_o^{(2)}(k_\rho \rho) \tilde{G}(k_\rho) \qquad (18)$$

where, G and $\tilde{G}$ are the spatial and frequency domain Green's functions respectively, $H_0^{(2)}$ is the Hankel function of the second kind and SIP is the Sommerfeld integral path. The Sommerfeld integral is a complex valued function with singularities and its integration path extends to infinity; hence, its numerical evaluation is difficult and time consuming. It has been shown that this difficulty can be overcome by approximating the frequency domain Green's functions in terms of complex exponentials of which Hankel transforms can be calculated analytically with the use of the following Sommerfeld identity [15]:

$$\frac{e^{-jkr}}{r} = -\frac{j}{2}\int_{SIP} dk_\rho k_\rho H_o^{(2)}(k_\rho \rho)\frac{e^{-jk_z|z|}}{k_z} \tag{19}$$

A complete set of closed-form spatial Green's functions are introduced by Dural et al. [10] for general sources and plenary layered media. Then, a new robust approach based on a two-level approximation has been proposed for derivation of closed-form Green's functions for plenary layered media by Aksun [11]. The most important step of this approach is the use of generalized pencil of function (GPOF) method for deriving the closed-form Green's functions [16]. This method is more robust and less noisy, and also provides a good measure for choosing the number of exponentials required in the approximation. In this study, frequency domain Green's functions are sampled across the path obtained by appropriately modifying the SIP shown in Figure 4, and in order to convert the spectral-domain Green's functions to spatial-domain by evaluating the Sommerfeld integral, a GPOF algorithm is used. The SIP is divided into two segments, Cap1 and Cap2, which have the following parametric equations:

For Cap1: $k_{z_i} = -jk_i\left[T_{o2} + t\right]$

For Cap2: $k_{z_i} = -jk_i\left[-jt + (1 - \frac{t}{T_{o2}})\right]$ (20)

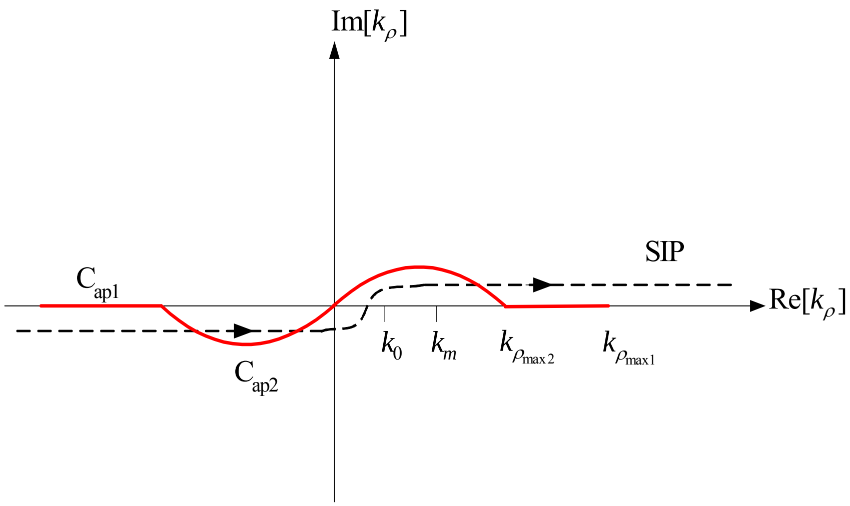


Fig.4. Sommerfeld integration path and integration path for two-level approximation [11]

After sampling the frequency domain Green's function except the 1/2jkzi term, GPOF method is used to obtain the following exponential approximation of the function:

$$\tilde{G} = \frac{1}{j2k_{z_i}} \left\{ \sum_{n=1}^{N_1} a_{1n} e^{-b_{1n}k_z} + \sum_{n=1}^{N_2} a_{2n} e^{-b_{2n}k_z} \right\} \tag{21}$$

where, a1n, a2n and b1n, b2n denote, the coefficients and exponentials obtained by applying the GPOF method to the first and second segments of the two level approach. After the frequency domain Green's functions are expressed in terms of sums of complex exponentials, each exponential in (21) is transformed into the spatial domain by using the Sommerfeld identity given in (19). The resulting spatial domain Green's functions are given in the following:

$$G \cong \sum_{n=1}^{N_1} a_{1n} \frac{e^{-jk_i r_{1n}}}{r_{1n}} + \sum_{n=1}^{N_2} a_{2n} \frac{e^{-jk_i r_{2n}}}{r_{2n}} \tag{22}$$

where, $\rho = \sqrt{x^2 + y^2}$, $r_{1n} = \sqrt{\rho^2 - b_{1n}^2}$, $r_{2n} = \sqrt{\rho^2 - b_{2n}^2}$ and ki is the wave number of the ith layer.

4 SCATTERING PARAMETER ANALYSIS

In order to obtain the scattering parameters, a two port transmission line is commonly used. For a two ports, generalized pencil of function (GPOF) procedure is applied to calculate the S-parameters. Having calculated the current densities on the conductors, the current on each port of the transmission line is written as a linear combination of exponentials as

$$I(l) \approx \sum_{i=1}^{N} I_i . e^{(\alpha_i + j\beta_i)l} \tag{23}$$

where αi and βi correspond to the attenuation and propagation constants of the ith mode of the current, respectively, and l is the distance along the port transmission line. The current can be expressed by two exponentials with complex coefficients corresponding to the incident and reflected waves at the corresponding ports. First, port1 is excited and current distributions are expressed as

$$\begin{aligned} I_1(l) &= I_{11}^{+} e^{-j\beta_1 l} + I_{11}^{-} e^{+j\beta_1 l} \\ I_2(l) &= I_{21}^{+} e^{-j\beta_2 l} + I_{21}^{-} e^{+j\beta_2 l} \end{aligned} \tag{24}$$

where Iij are the current wave coefficient on the ith port transmission line when port j is excited. These current wave coefficients can be related to each other by using S-parameter matrix

$$\begin{bmatrix} -I_{11}^{-} \\ I_{21}^{-} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} I_{11}^{+} \\ -I_{21}^{-} \end{bmatrix} \tag{25}$$

Then port2 is excited and current distribution on the entire structure is again solved and the port transmission line current densities are expressed as

$$\begin{aligned} I_1(l) &= I_{12}^{+} e^{-j\beta_1 l} + I_{12}^{-} e^{+j\beta_1 l} \\ I_2(l) &= I_{22}^{+} e^{-j\beta_2 l} + I_{22}^{-} e^{+j\beta_2 l} \end{aligned} \tag{26}$$

Similarly, these current wave coefficients can be written in terms of S-parameters as

$$\begin{bmatrix} -I_{12}^{-} \\ I_{22}^{-} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} I_{12}^{+} \\ -I_{22}^{-} \end{bmatrix} \tag{27}$$

Since the end of approximations it is found that αi's are small compared to βi's αi's are not written in the above equations. In order to find the S-parameters for a general two port network, following the matrix equation, that is elated with (24) and (25), is used.

$$\begin{bmatrix} I_{11}^{+} & I_{12}^{+} & 0 & 0 \\ 0 & 0 & I_{11}^{+} & I_{12}^{+} \\ I_{21}^{+} & I_{22}^{+} & 0 & 0 \\ 0 & 0 & I_{21}^{+} & I_{22}^{+} \end{bmatrix} \begin{bmatrix} S_{11} \\ S_{12} \\ S_{21} \\ S_{22} \end{bmatrix} = \begin{bmatrix} -I_{11}^{-} \\ -I_{12}^{-} \\ -I_{21}^{-} \\ -I_{22}^{-} \end{bmatrix} \tag{28}$$

Finally, the S-parameters obtained are converted to the S-parameters with the reference impedance of 50Ω. It is necessary to know the characteristic impedances of the port transmission lines to convert generalized S-parameters to normalized ones. Due to the exact calculation of the characteristic impedance of a microstrip line in a layered medium is quite time consuming, a method based on a quasi-TEM approach is used in this article. This method provides analytical expressions for the characteristic impedance of a microstrip line and a stripline [17].

5 SIMULATION ALGORITHM

A simplified flowchart of the algorithm according to the MPIE-MoM solution method described in the previous sections is given in Figure 5. The software starts by reading the layout file that includes the operating frequency, layer information, meshing parameters, and port definitions. According to the meshing parameters, the geometry is subdivided and number of unknown is determined. After calculating the coordinates of the basis and test functions, similarities among the inner product terms are tabulated in order to assist the computation in the further steps. Then MoM matrix is filled using the basis functions and Green's functions. The resulting linear system is solved for the unknown basis amplitudes. Finally, circuit parameters are saved.

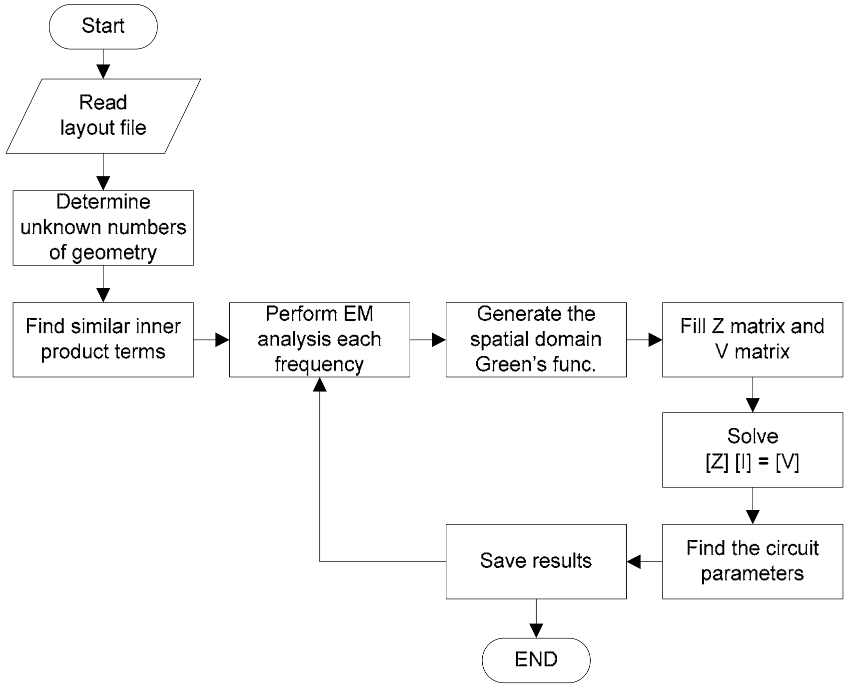
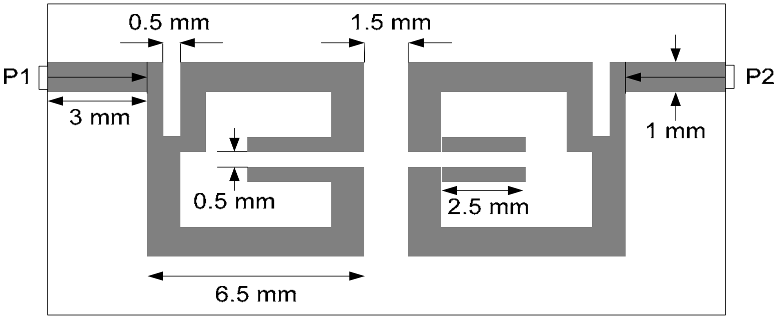


Fig.5. Simple flowchart of the algorithm

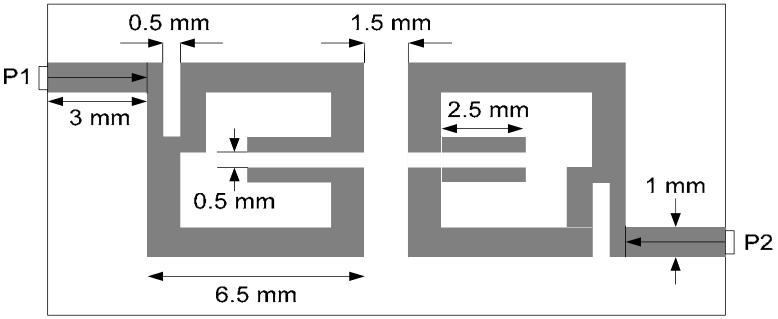
6 NUMERICAL EXAMPLES

In this section, the MPIE-MoM technique is applied to microwave filter configuration consists of coupled microstrip open-loop resonators. For the examples, general microstrip geometry in a layered media is assumed where all layers and the ground plane extend to infinity in the horizontal plane, and the conductors are lossless and infinitesimally thin. The S-parameters provided here are normalized with respect to 50-Ω reference impedance. The results are compared with the well-known commercially available full-wave EM Simulator Sonnet.

Figure 6 shows the analyzed dual-mod microwave planar filter configuration [18,19]. First example is the dual-mode linear phase filter as shown in Figure 6(a) and second example is the dual-mode elliptic filter as shown Figure 6(b). These filters have been constructed from same type of resonators by exchanging feed lines as cross and diagonally. The filters consist of a set of microstrip coupled open-loop resonators with a spacing of 1.5mm and an open-gap of 0.5mm on a substrate with a thickness of 1.27mm and εr =10.2. The size of open-loop arms is 1mm and the length of the feed line is 3mm. The filters were analyzed by using the MPIE-MoM technique, over a frequency of 2.0 GHz to 3.0 GHz.



(a)



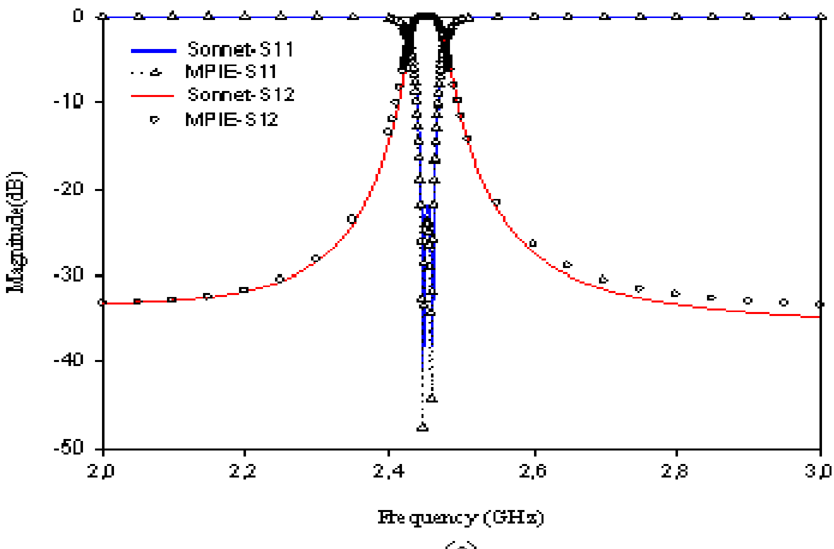
(b)

Fig.6. Geometry of dual-mode filters configuration. (a) Dual-mode linear phase filter, (b) Dual-mode elliptic filter

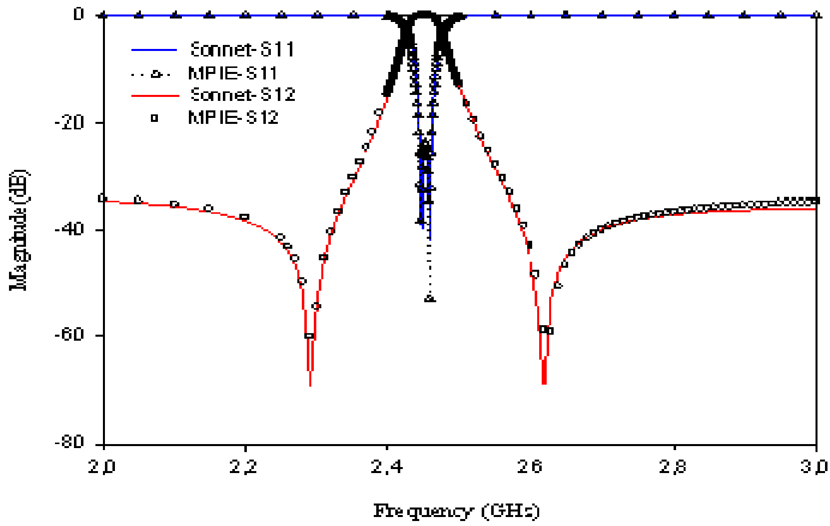
The geometries are a symmetric structure so S11= S22 and S12= S21. S11 and S12 of the filters are obtained using the MPIE-MoM technique and compared to the results of a commercial software package called "em" by Sonnet Software, Inc [20]. Magnitudes of S11 and S12 of the dual-mode linear phase filter and the dual-mode elliptic filter shown in Figure 7(a) and 7(b), respectively. There is a good agreement between results of the MPIE-MoM technique and the commercial software package for magnitude of S-parameters. Both filters represent band-pass filter characteristics, when seeing frequency response of filters.

7 CONCLUSIONS

We have presented a numerically efficient MPIE-MoM technique for analysis of dual-mode filter. The method of moments is applied to the solution of the mixed potential integral equation in the spatial-domain in conjunction with the closed-form Green's functions. Microwave filters consist of coupled microstrip open-loop resonators are analyzed to demonstrate the efficiency and accuracy of MPIE-MoM technique. The results obtained are in good agreement with the results obtained from well-known EM software SONNET. The main advantage of MPIE-MoM technique is its generality for open geometries and it allows a large variety of microstrip structures to be characterized with high accuracy and efficiency



(a)



(b)

Fig.7. Magnitudes of S11 and S12 of the dual-mode linear phase filter and the dual-mode elliptic filter

APPENDIX A-GENERALIZED PENCIL OF FUNCTION ALGORITHM

The generalized pencil of function method is used to estimate the poles of an EM system from its transient response [16,21]. The generalized pencil of function (GPOF) algorithm is used to approximate the spectral domain Green's functions with complex exponentials.

Consider an EM transient signal which can be approximated as follows

$$y_k = \sum_{i=1}^{M} b_i . e^{s_i \delta t k} \qquad k = 0,1,...,N-1 \tag{A.1}$$

where bi is the complex residues, si are the complex poles, and δt is the sampling interval. In order to find the poles, one can use the following algorithm [16,21]:

i) Construct the following matrices,

$$Y_1 = \left[y_0, y_1, \cdots, y_{L-1} \right] \tag{A.2}$$

$$Y_2 = \left[y_1, y_2, \cdots, y_L \right] \tag{A.3}$$

$$y_i = \left[y_i , y_{i+1}, \cdots, y_{i+N-L-1} \right]^T \tag{A.4}$$

and L is the pencil parameter, and its optimal choice is around L = N/2 [16,21].

ii) Find a Z matrix as follows,

$$\begin{aligned} V\, D^{-1}\, U^H &= SVD(Y_1) \\ V &\leftarrow [V]_{M\times M} \\ U &\leftarrow [U]_{M\times M} \\ D &\leftarrow [D]_{M\times M} \end{aligned} \tag{A.5}$$

$$Z = D^{-1} U^H Y_2 V \tag{A.6}$$

where SVD(·) and superscript H denote the singular value decomposition process and the complex conjugate transpose of a matrix, respectively. The number of exponentials, M, is selected according to the significant singular values of the matrix Y1.

iii) The poles of the system are obtained as

$$s_i = \frac{\log z_i}{\delta t} \qquad i = 1, 2, \cdots, M \tag{A.7}$$

where zi's are the eigen values of the Z matrix evaluated in step ii.

iv) The residues are found from the least-squares solution of the following system

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_m \\ \vdots & \vdots & \cdots & \vdots \\ z_1^{N-1} & z_2^{N-1} & \cdots & z_M^{N-1} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_M \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-1} \end{bmatrix} \tag{A.8}$$

REFERENCES

[1] R. Sorrentino, Ed., Numerical Methods for Passive Microwave and Millimeter Wave Structures. New York:IEEE Press, 1989.

[2] T. Rozzi and M. Mongiardo, Open Electromagnetic Waveguides. London, U.K.: IEE Press, 1997.

[3] J. R. Mosig and F. Gardiol, "General integral equation formulation for microstrip antennas and scatters," Ins. Elect. Eng. Proc., pt. H, vol. 132, pp. 424-432, Dec. 1985.

[4] J. R. Mosig, "Arbitrary Shaped Microstrip structures and their analysis with a mixed potential integral equation," IEEE Trans. Microwave Theory Tech., vol. 36, pp. 314-323, Feb. 1988.

[5] Y. L. Chow, N. Hojjat, S. Safavi-Naeini, and R. Faraji-Dana, "Spectral Green's functions for multilayer media in a convenient computational form," IEE Proc.-Microw. Antennas Propag., vol. 145, pp. 85-91, Feb. 1998.

[6] M. Vrancken and Guy A. E. Vandenbosch, "Hybrid dyadic-mixed-potential and combined spectral-space domain integral-equation analysis of quasi-3-D structures in Stratified Media," IEEE Trans. Microwave Theory Tech., vol. 51, pp. 216-225, Jan. 2003.

[7] M. E. Yavuz, Development of an Electromagnetic Simulation Algorithm for Planar Geometries with Multiple Vertical Conductors, M. S. thesis, Electrical and Computer Engineering, University of Koç, Istanbul, Turkey, 2003.

[8] N. V. Shuley, R. R. Boix, F. Medina and M. Horno, "On the fast approximation of Green' functions in MPIE formulations for planar layered media," IEEE Trans. Microwave Theory Tech., vol. 50, pp. 2185-2192, Sep. 2002.

[9] R. F. Harrington, Field Computation by Method of Moment Methods, Melbourne, FL: Krieger, 1983.

[10] G. Dural and M. I. Aksun, "Closed-form Green's functions for general sources and stratified media," IEEE Trans. Microwave Theory Tech., vol. 43, pp. 1545-1552, July 1995.

[11] M. I. Aksun, "A robust approach for the derivation of closed-form Green's functions", IEEE Trans. Microwave Theory Tech., vol. 44, pp. 651-658, May 1996.

[12] M. I. Aksun, R. Mittra, "Choice of expansion and testing functions for the method of moments applied to a class of electromagnetic Problems," IEEE Trans. Microwave Theory Tech., vol. 41, No.3, pp. 503-509, March 1993.

[13] L. Alatan, M. I. Aksun, K. Mahadevan and M. T. Birand, "Analytical evaluation of the MoM matrix elements," IEEE Trans. Microwave Theory Tech., vol. 44, pp. 519-525, April 1996.

[14] L. Alatan, M. I. Aksun, K. Leblebicioğlu and M. T. Birand, "Use of computationally efficient method of moments in the optimization of printed antennas," IEEE Trans. Antennas Propagat., vol. 47, No. 4, pp. 725-732, April 1999.

[15] Y. L. Chow, J. J. Yang, D. F. Fang and G. E. Howard, “Closed-form spatial Green’s function for the thick substrate,” IEEE Trans. Microwave Theory Tech., vol. 39, pp. 588-592, Mar. 1991.

[16] Y. Hua and T. K. Sarkar, “Generalized pencil-of-function method for extracting poles of an EM system from its transient Response,” IEEE Trans. Antennas Propagat., vol. 37, pp. 229-234, Feb. 1989.

[17] F. Gardiol, Microstrip Circuits. Wiley, New York, 1994

[18] Z. Genc and H. H. Balik, “Numerically efficient MPIE-MoM technique for analysis of microstrip structures in layered media,” German Microwave Conference-GEMIC 2006, March 2006, Germany.

[19] G. Kaptan, C. Karpuz, A. Gorur, “Microwave dual-mode planar filter design for mobile communication systems,” Information Technologies Conference III-BILGITEK, Oct 2004, Denizli, Turkey.

[20] SONNET SOFWARE INC., User’s Manuel Release 10, November 2004, USA.

[21] Noyan Kınayman, A Novel CAD Algorithm for The Analysis of Printed Geometries, PhD thesis, Electrical and Electronics Engineering and The Institute of Engineering and Science of Bilkent University, Turkey, 1997.