Journal of Science and Technology
2 (2), 2008, 177-193

A NEW APPROACH TO ANALYSIS OF RECTANGULAR WAVEGUIDES

Yavuz Erol[1] and Hasan H. Balik[2] *
[1]University of Firat, Department of Electrical and Electronics Engineering, Elazig, Turkey, yavuzerol23@gmail.com
[2]Beykent University, Department of Electronics and Communication Engineering, Istanbul, Turkey, hasanbalik@gmail.com
*Corresponding Author

Received: 15 May 2008, Accepted: 16 July 2008

ÖZET

FDTD metodu dalga kılavuzu modellemede yaygın olarak kullanılan güçlü sayısal teknikler arasındadır. Bu yayında 2 boyutlu FDTD algoritması yardımıyla dikdörtgen kesitli dalga kılavuzu analizine farklı bir bakış açısı getirilmiştir. TM ve TE modları için iki ayrı simülasyon yapılarak dalga kılavuzunun zaman cevabı elde edilmiştir. Kılavuz içerisine farklı dielektrik malzemeler yerleştirilerek etkileri araştırılmıştır. WR284 adlı dalga kılavuzunun frekans cevabı, hızlı fourier dönüşümü (FFT) ve sıfır-ekleme teknikleri kullanılarak bulunmuştur. Frekans cevabı üzerinde görülen rezonans frekanslarının kılavuzun mod kesim frekanslarına denk geldiği ve FDTD simülasyon sonuçlarının analitik sonuçlar ile uyum içerisinde olduğu gösterilmiştir.

Anahtar Kelimeler: *2-D FDTD; FFT; dikdörtgen kesitli dalga kılavuzu, mod kesim frekansı*

ABSTRACT.

FDTD is one of the strong numerical techniques commonly used to model waveguides. In this paper, a new approach is presented to analyze rectangular waveguides by using 2-D FDTD algorithm. Two time response of the waveguide for TM and TE modes by using tow different simulation. The effects of the dielectrics have also been observed. The frequency response of the waveguide called WR284 has been found by using FFT and zero adding techniques. The FDTD simulation results for TE and TM modes are compared with analytical results and shown to have good agreement to analytical results.

Keywords*: 2-D FDTD; FFT; rectangular waveguide; mode cut-off frequency*

1 INTRODUCTION

Rectangular waveguide is one of the earliest type of the transmission lines and still commonly used in many current applications. A lot of components such as isolators, detectors, attenuators, couplers and slotted lines are available to use for various standard waveguide bands between 1 GHz to above 220 GHz [3]. Most of the early numerical analysis methods for microwave integrated circuits were based on either quasi-static or magnetic wall approximations, in which only the TEM-mode of propagation was taken into account. It becomes necessary to use full-wave analysis to include effects like coupling due to surface waves and discontinuities at operating high frequencies to fulfil contemporary demands for spectrum space,. At the operating frequencies where these waveguides commonly used, the assumptions which are valid only at low frequencies can not be applied to gain accurate results. Therefore full-wave analysis techniques must be required. There are many full-wave numerical techniques to solve electromagnetic problems. Some techniques solve the problem in the time domain [1,8,10] whereas others solve in frequency domain [4-6]. Although full-wave numerical technique gives accurate results, it requires more time and computer resources for solutions. The demands of the design engineer require a technique which is accurate, yet retains the interactive design capabilities of the simpler techniques.

The finite-difference time-domain (FDTD) method has been widely used as an efficient tool for the accurate solving of a great variety of electromagnetic problems. The FDTD method has been successfully applied to scattering problems [14,18], extended to the study of microstrip discontinuities [15] and has recently been used to analyze resonant devices [19]. It has also found widespread applications in the area of microwave devices and guiding structures, such as waveguides, resonators, junctions, microstrips, vias, interconnects, and transmission lines [9,11,16,17]. Techniques have been developed techniques to improve the efficiency of the FDTD method in the modelling of these devices and structures. Such techniques include the compact 2-D FDTD method for guided wave structures, and the absorbing boundary conditions specifically developed for guided wave problems [7].

In this contribution, rectangular waveguide called WR284 has been numerically analyzed by introduced 2-D FDTD method to show the accuracy. The frequency response of waveguide is obtained using Fast Fourier Transform and zero padding techniques. It has been shown that the introduced technique is accurate to find mode cut-off frequency.

2 REVIEWS TO RECTANGULAR WAVEGUIDES

The presence of magnetic or electric field components along the propagation direction requires that the wave field components vary with position along the planar phase fronts. Such waves are called transverse electric (TE) or transverse magnetic (TM) waves respectively. TE and TM modes are the only types of electromagnetic waves that can propagate in metallic tube waveguides which confine the wave energy in both of the transverse directions. Therefore the hollow rectangular waveguide can propagate TM and TE modes, but not TEM waves.

A rectangular waveguide with internal dimensions a, and b is illustrated in Figure 1 with Cartesian co-ordinate system to be used in the analysis of the electromagnetic wave propagation inside the tube. It will be assumed that the waveguide is of uniform cross section with perfectly conducting walls and infinite length in the z direction, the direction of propagation of the wave.

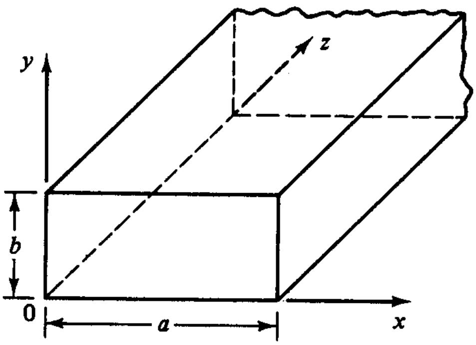

Fig. 1. Dimensions of rectangular waveguides

In practice the boundary consists of a high conductivity metal such as copper or silver, although brass or aluminium is also used. A waveguide with a rectangular cross section is the preferred shape for most applications. By expanding Maxwell's equations in a rectangular co-ordinate system and rearranging, it is shown that each of the field components Ex, Ey, Hx and Hy can be expressed in terms of longitudinally directed components Ez and Hz.

Two independent wave equations are derived for Ez and Hz showing that the analysis can be divided into two basic types or modes of propagating waves. In the first case, the magnetic field is in a plane normal to the direction of propagation but has a component of electric field in the direction of propagation. This mode of propagation is called transverse magnetic or TM mode (for which Hz=0 and Ez≠0). Wave equation for Ez is shown in equation (1).

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + k_c^2 E_z = 0 \tag{1}$$

Expressions for the E and H fields of TM modes in a rectangular waveguide are given below

$$E_z = E_0 \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{j(\omega t-\beta z)} \tag{2}$$

$$E_x = -\frac{j\beta E_0}{k^2}\frac{m\pi}{a}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{j(\omega t-\beta z)} \tag{3}$$

$$E_y = -\frac{j\beta E_0}{k^2}\frac{n\pi}{b}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{j(\omega t-\beta z)} \tag{4}$$

$$H_x = \frac{j\omega\varepsilon E_0}{k^2}\frac{n\pi}{b}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{j(\omega t-\beta z)} \tag{5}$$

$$H_y = -\frac{j\omega\varepsilon E_0}{k^2}\frac{m\pi}{a}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{j(\omega t-\beta z)} \tag{6}$$

In the second case the electric field is entirely transverse but there is an axial component of magnetic field. This mode of propagation is called transverse electric or TE mode (for which Ez=0 and Hz≠0). Wave equation for Hz is shown in equation (7).

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + k_c^2 H_z = 0 \tag{7}$$

Expressions for the E and H fields of TE modes in a rectangular waveguide are given below

$$H_z = H_0 \cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{j(\omega t-\beta z)} \tag{8}$$

$$E_x = \frac{j\omega\mu H_0}{k^2}\frac{n\pi}{b}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{j(\omega t-\beta z)} \tag{9}$$

$$E_y = -\frac{j\omega\mu H_0}{k^2}\frac{m\pi}{a}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{j(\omega t-\beta z)} \tag{10}$$

$$H_x = \frac{j\beta H_0}{k^2}\frac{m\pi}{a}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{j(\omega t-\beta z)} \tag{11}$$

$$H_y = \frac{j\beta H_0}{k^2}\frac{n\pi}{b}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{j(\omega t-\beta z)} \tag{12}$$

The propagation constant in z direction is shown equation (13),

$$\gamma_{mn} = \sqrt{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] - \left[\omega^2\mu\varepsilon\right]} \tag{13}$$

Equation (13) defines an infinite number of modes dependent on the values given to m and n. However, only a finite number will be transmission modes since in this case γ must be purely imaginary and so the frequency must be high enough. The mode frequency must be higher than cut-off frequency (fc) so that electromagnetic waves can propagate trough waveguides. If mode frequency less than cut-off frequency, electromagnetic waves decay rapidly in the direction of waveguide axes. There is infinite number of modes every which has different cut-off frequency from others in the waveguide.

The mode cut-off frequency of rectangular waveguide is calculated by (14), where m and n are mode degrees.

$$f_{c,mn} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \tag{14}$$

3 2-D FDTD METHOD

In 1966, K. S. Yee [12] proposed a novel technique to solve Maxwell's equations for initial boundary problems in the time domain. The original FDTD algorithm is specially designed to solve electromagnetic problems in a rectangular coordinate system and it assumes a lossless, isotropic and non-dispersive medium. With FDTD, it is possible to analyze EM problems directly in time domain for a broad range of frequencies, only in one simulation [13]. Finite-difference approximations of differential equations have been extensively used not only for the solution of Maxwell's equations both in the time and in the frequency domain but for a large variety of applications in physics. The FDTD method uses Maxwell's equations that define the propagation of an electromagnetic wave and the relationship between the electric and magnetic fields.

$$\vec{\nabla} \times \vec{E} = -\mu\frac{\partial\vec{H}}{\partial t} \tag{15}$$

$$\vec{\nabla} \times \vec{H} = \varepsilon\frac{\partial\vec{E}}{\partial t} + \sigma\vec{E} \tag{16}$$

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon} \tag{17}$$

$$\vec{\nabla}.\vec{H} = 0 \tag{18}$$

FDTD is a direct full-wave numerical solution of Maxwell's time-dependent curl equations. The time and space derivatives of the partial differential equations can be approximated in three different schemes: forward, backward and central. It applies simple, second order accurate central difference approximations for the space and time derivatives of the electric and magnetic fields directly to the respective differential operators of the curl equations.

For the scalar field F, the central difference expressions for the time and space derivatives are:

$$\frac{\partial F^n(i,j,k)}{\partial t} = \frac{F^{n+1/2}(i,j,k) - F^{n-1/2}(i,j,k)}{\Delta t} + O\left[(\Delta t)^2\right] \tag{19}$$

$$\frac{\partial F^n(i,j,k)}{\partial x} = \frac{F^n(i+1/2,j,k) - F^n(i-1/2,j,k)}{\Delta x} + O\left[(\Delta x)^2\right] \tag{20}$$

Where O(Δx)2 and O(Δt)2 terms are errors which occurred while numerical derivation by central finite differences approximation. At the numerical derivation that has second order accuracy, these errors can generally be ignored because they are too small [2]. Fn is the field value at time nΔt, Fn+1/2 is the value at nΔt+Δt/2 and Fn-1/2 is the value at nΔt-Δt/2. The superscript n indicates the time step.

As shown in Figure 2, the components of E and H are evaluated and calculated at alternate half-time steps which are called as leap-frog computation. Electric and magnetic fields are calculated at time instants t=0, Δt, 2Δt, 3Δt, … and t=1/2Δt, 3/2Δt, 5/2Δt, … respectively.

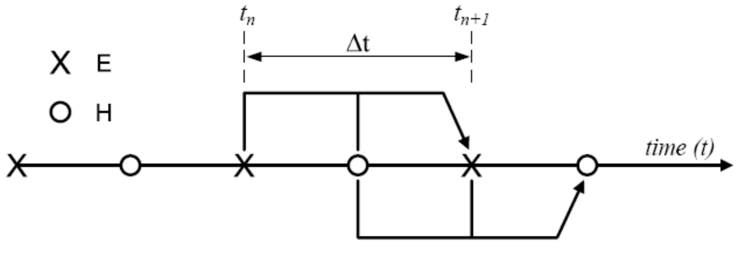

Fig. 2. The time flow graph of field updating in Yee Algorithm

E and H fields are assumed interleaved around a cell whose origin is at the location i, j, and k. Every E field is located 1/2 cell width from the origin in the direction of its orientation; every H field is offset 1/2 cell in each direction except that of its orientation.

3.1. FORMULATION OF TM MODE

In 2-D FDTD analysis, it is assumed that there are no changes in z direction. In other words δ/δz assumed to be zero [11]. In this case TM mode equations for homogeneous and lossless domain (σ=0) can be found as shown in (21), (22) and (23).

$$\frac{\partial H_x}{\partial t} = -\frac{1}{\mu}\frac{\partial E_z}{\partial y} \tag{21}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\frac{\partial E_z}{\partial x} \tag{22}$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) \tag{23}$$

In TM mode, Hx, Hy and Ez field components are placed on x-y plane as shown in Figure 3. The inside walls of waveguide coated with perfectly electrical conductor (PEC). The boundary condition for a perfect electric conductor requires the tangential E field to be zero at the boundary.

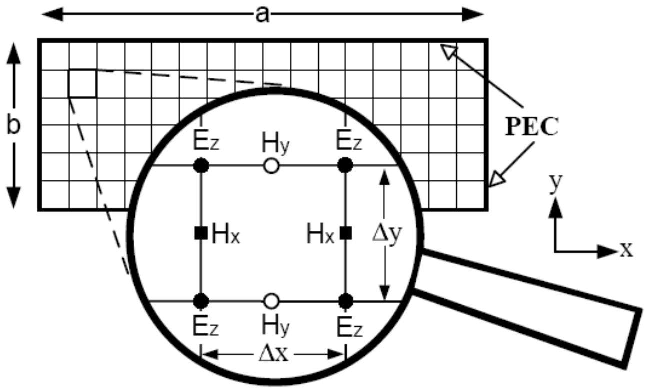

Fig. 3. Position of field components in TM mode

When equations of (21), (22) and (23) are discredited with central difference approximation, FDTD equations for TM mode are obtained as (24), (25) and (26).

$$H_x^{n+1/2}(i,j) = H_x^{n-1/2}(i,j) - \frac{\Delta t}{\mu\Delta y}\left[E_z^n(i,j+1/2) - E_z^n(i,j-1/2)\right] \tag{24}$$

$$H_y^{n+1/2}(i,j) = H_y^{n-1/2}(i,j) + \frac{\Delta t}{\mu\Delta x}\left[E_z^n(i+1/2,j) - E_z^n(i-1/2,j)\right] \tag{25}$$

$$E_z^{n+1}(i,j) = E_z^{n}(i,j) + \frac{\Delta t}{\varepsilon \Delta x}\left[H_y^{n+1/2}(i+1/2,j) - H_y^{n+1/2}(i-1/2,j)\right] - \frac{\Delta t}{\varepsilon \Delta y}\left[H_x^{n+1/2}(i,j+1/2) - H_x^{n+1/2}(i,j-1/2)\right] \tag{26}$$

During FDTD simulation of TM mode Hx, Hy and Ez field components are calculated iteratively.

3.2. FORMULATION OF TE MODE

TE mode equations for homogeneous and lossless domain can be found as shown in (27), (28) and (29).

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\frac{\partial H_z}{\partial y} \tag{27}$$

$$\frac{\partial E_y}{\partial t} = -\frac{1}{\varepsilon}\frac{\partial H_z}{\partial x} \tag{28}$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) \tag{29}$$

In TE mode, Ex, Ey and Hz field components are placed on x-y plane as shown in Figure 4.

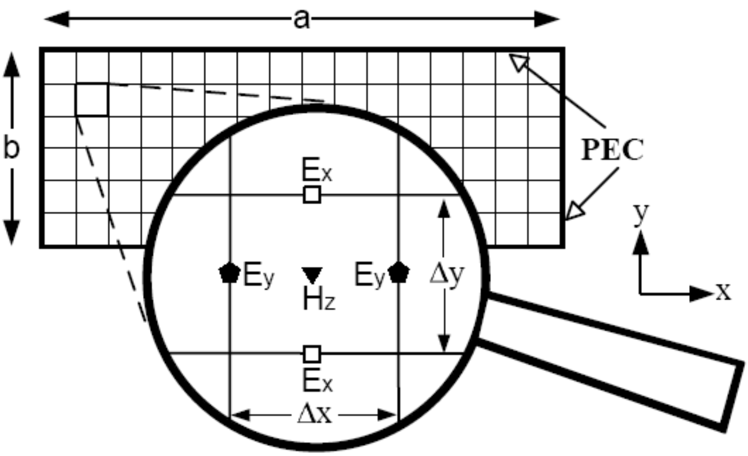

Fig. 4. Position of field components in TE mode

When equations of (27), (28) and (29) are discredited with central difference approach, FDTD equations for TE mode are obtained as (30), (31) and (32).

$$E_x^{n+1/2}(i,j) = E_x^{n-1/2}(i,j) + \frac{\Delta t}{\varepsilon \Delta y}\left[H_z^n(i,j+1/2) - H_z^n(i,j-1/2)\right] \tag{30}$$

$$E_y^{n+1/2}(i,j) = E_y^{n-1/2}(i,j) - \frac{\Delta t}{\varepsilon \Delta x}\left[H_z^n(i+1/2,j) - H_z^n(i-1/2,j)\right] \tag{31}$$

$$\begin{aligned} H_z^{n+1}(i,j) = H_z^n(i,j) &+ \frac{\Delta t}{\mu \Delta y}\left[E_x^{n+1/2}(i,j+1/2) - E_x^{n+1/2}(i,j-1/2)\right] \\ &- \frac{\Delta t}{\mu \Delta x}\left[E_y^{n+1/2}(i+1/2,j) - E_y^{n+1/2}(i-1/2,j)\right] \end{aligned} \tag{32}$$

Similarly during simulation of TE mode Ex, Ey and Hz field components are calculated iteratively.

4 COMPUTER SIMULATION AND NUMERICAL RESULTS

First the analyzed mode and then max frequency (fmax) must be determined. Because this process is necessary to specify time step and cell size of FDTD simulation. The ratio of minimum wavelength (c/fmax) and cell size must be chosen appropriately to avoid numerical dispersion. This choice is crucial. Time step (Δt) is determined by cell size according to Courant stability criteria.

$$\Delta t \le \frac{1}{(c/\sqrt{\varepsilon_r}) \cdot \sqrt{(1/\Delta x)^2 + (1/\Delta y)^2}} \tag{33}$$

Therefore waveguide has become divided into (Nx) ×(Ny) number of cells so that Δx and Δy are cell sizes in x and y axes respectively. As a source, Gauss pulse of which durations are chosen according to maximum frequency used. Inside the FDTD loop, Gauss pulse is applied at one point, and the calculated field's components of observation point are saved. So the time response of rectangular waveguides is gained. Fourier transform of time response are taken to calculate frequency response of rectangular waveguide.

The propagation of Gauss pulse in the rectangular waveguide has been shown in Figure 5. It has also shown how the waves reflect from the closed PEC walls when time step increased.

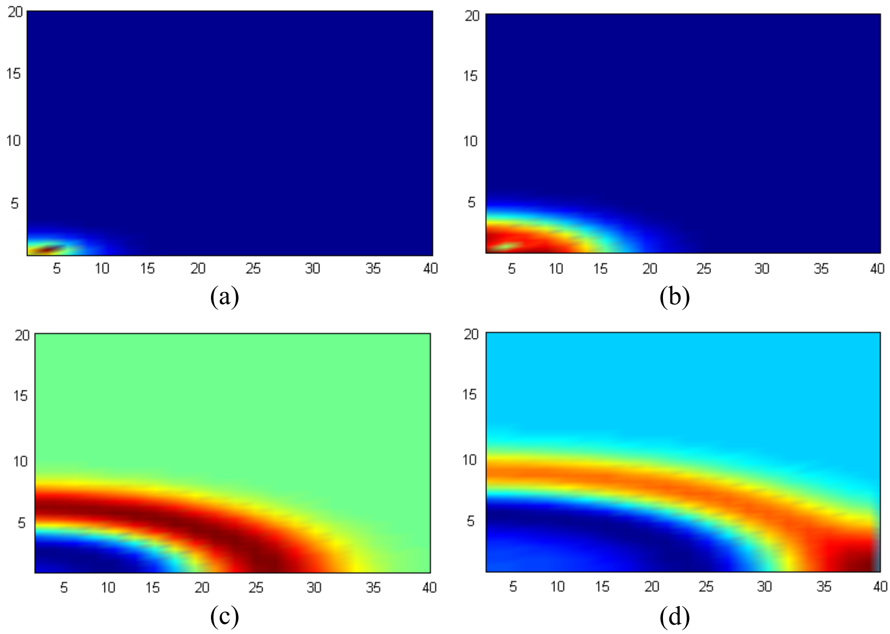

Fig. 5. Display of Hz field generated by FDTD program at time steps (a) 10, (b) 20, (c) 40 and (d) 48

In this paper WR284 of which dimensions 72.136 mm x 34.036 mm have been analyzed by enhance 2D-FDTD. These waveguides are used at S band (2.6 GHz-3.95 GHz). Dominant mode of WR284 is TE10 mode at 2.079 GHz.

4.1. TM MODE ANALYSIS

4.1.1. AIR-FILLED RECTANGULAR WAVEGUIDE

Chosen parameters used throughout the computer simulation of TM mode by 2D-FDTD technique are given in Table 1.

Table 1: 2D-FDTD parameters for TM modes

Gauss pulse width	165 ps
Δx (cell size in x-axes)	1.8034 mm
Δy (cell size in y-axes)	1.7018 mm
Nx (number of cell in x-axes)	40
Ny (number of cell in y-axes)	20
Δt (time step)	3 ps
T (simulation duration)	10 000Δt

30 000 number of zeros is added at the end of the data to increase frequency resolution, because FDTD simulation is terminated at certain time. Therefore detailed frequency response of the waveguide can be obtained by not extending the simulation so long.

Time and frequency responses of TM mode are given in Figure 6. Every peak shown in the frequency response at 0-8 GHz represents mode cut-off frequency. As shown in the frequency response, there is no TM mode in or before S band.

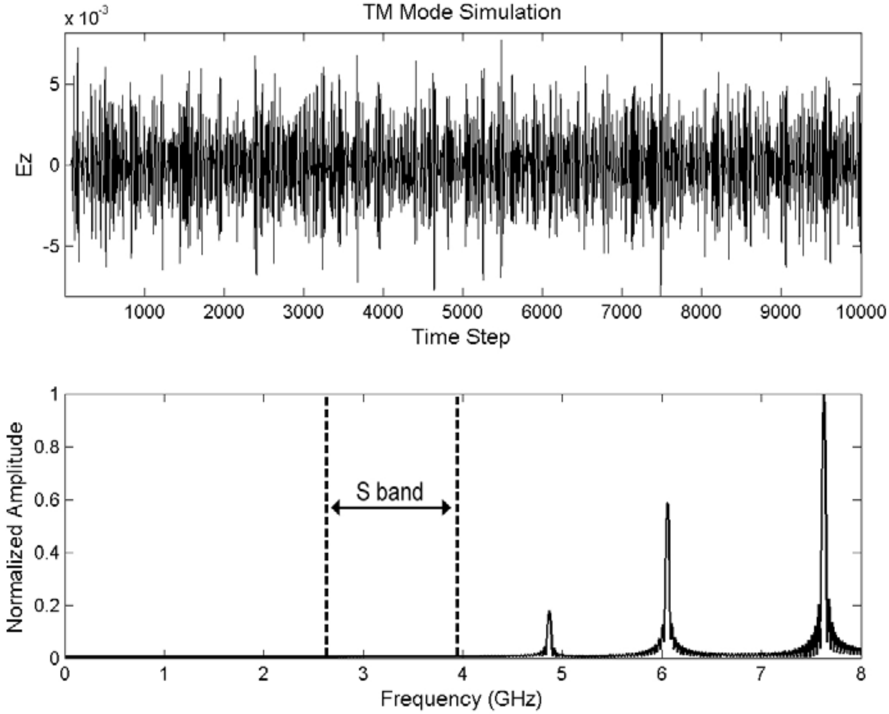

Fig. 6. Time and frequency responses for TM mode

4.1.2. DIELECTRIC FILLED RECTANGULAR WAVEGUIDE

Dielectric filled metallic waveguide has the same property as the air-filled one with the only exception that the cut-off frequency for the modes is lower due to the presence of the dielectric.

In Figure 7, the frequency response of the waveguide which is loaded with dielectric whose relative permittivity is 1.0 and 2.32 respectively has been shown.

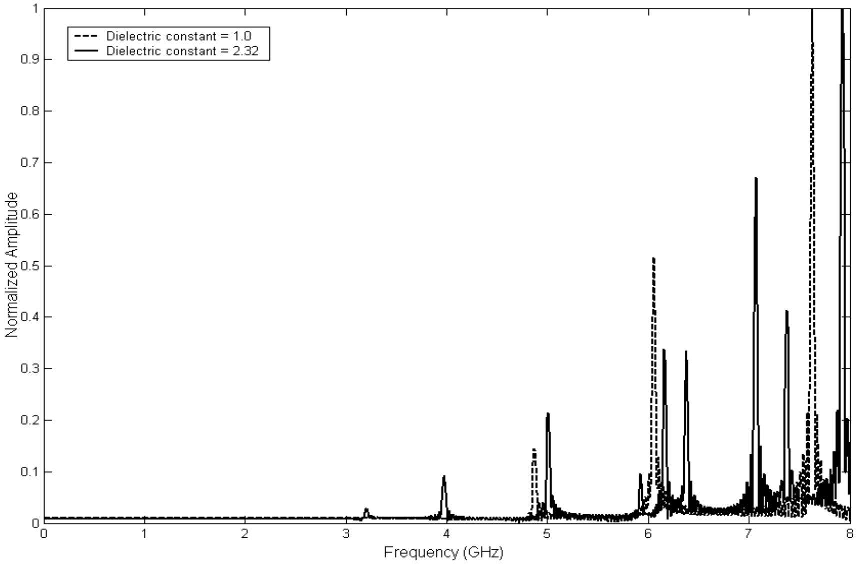

Fig. 7. 2 Frequency responses for different dielectric (TM mode)

Table 2 compares FDTD results and analytical results. In Table 2, cut-off frequencies of first 7 modes with least frequencies are given. It is demonstrated that the FDTD algorithm presented here is in good agreement with analytical results and error is around 0.1%.

4.2. TE MODE ANALYSIS

4.2.1. AIR-FILLED RECTANGULAR WAVEGUIDE

Chosen parameters used throughout the computer simulation of TE mode by 2D-FDTD technique are given in Table 3.

Table 3: 2D-FDTD parameters for TE modes

Gauss pulse width	110 ps
Δx (cell size in x-axes)	1.8034 mm
Δy (cell size in y-axes)	1.7018 mm
Nx (number of cell in x-axes)	40
Ny (number of cell in y-axes)	20
Δt (time step)	3 ps
T (simulation duration)	10 000Δt

Time and frequency response of TE modes are given in Figure 8. There is only one resonant frequency before S band as shown in Frequency response. This resonant frequency which is dominant mode as shown in the analytical results is equal to 2.079 GHz

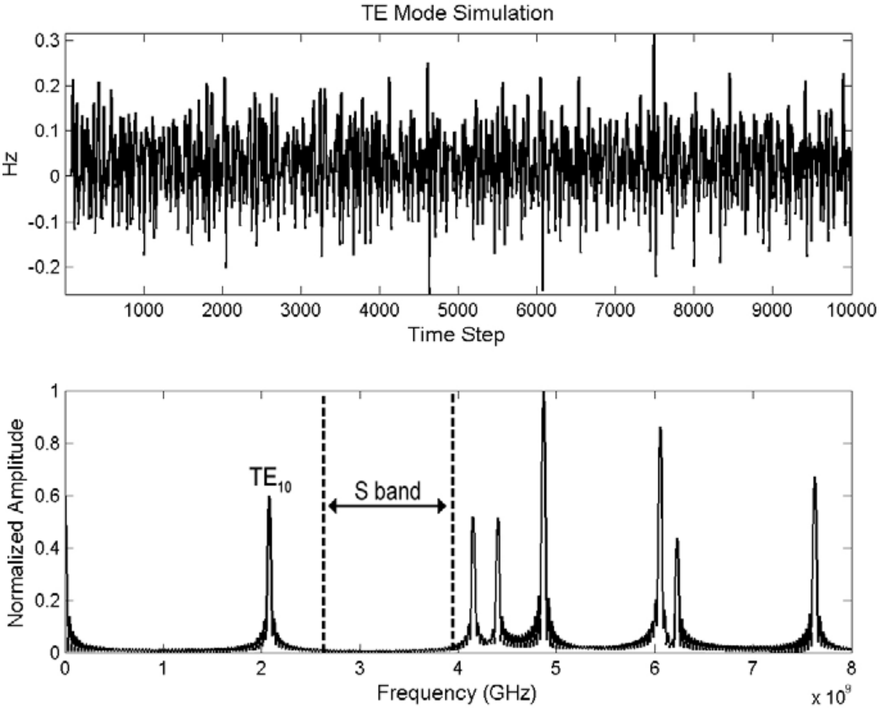

Fig. 8. Time and frequency responses for TE mode

4.2.2. DIELECTRIC FILLED RECTANGULAR WAVEGUIDE

In Figure 9, the frequency response of the waveguide that is loaded with dielectric whose relative permittivity is 1.0 and 2.32 has been shown.

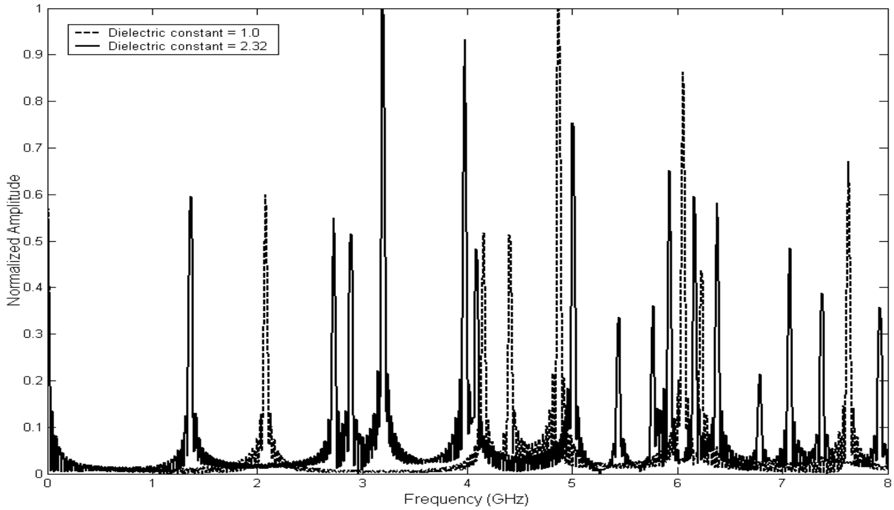

Fig. 9. 2 Frequency responses for different dielectric (TE mode)

Table 4 compares FDTD results and analytical results. It is demonstrated that FDTD algorithm presented here is good agreement and error is around 0.2%.

Table 4: Mode cut-off frequencies and FDTD simulation results

	εr =1.0			εr =2.32		
Modes	Exact	FDTD	Diff. %	Exact	FDTD	Diff. %
TE10	2.0794	2.0766	0.134	1.3652	1.3669	0.124
TE20	4.1588	4.1492	0.230	2.7304	2.7218	0.314
TE01	4.4071	4.3992	0.179	2.8934	2.8911	0.079
TE11	4.8730	4.8669	0.125	3.1993	3.1976	0.053
TE21	6.0596	6.0524	0.118	3.9783	3.9718	0.163
TE30	6.2382	6.2218	0.262	4.0956	4.0847	0.266
TE31	7.6379	7.6250	0.168	5.0145	5.0040	0.209

In Figure 10, there are cut-off frequencies of TE10, TE11, TM21 and TM31 modes for different relative permittivity. It has shown that mode cut-off frequency decreases when dielectric constant increases.

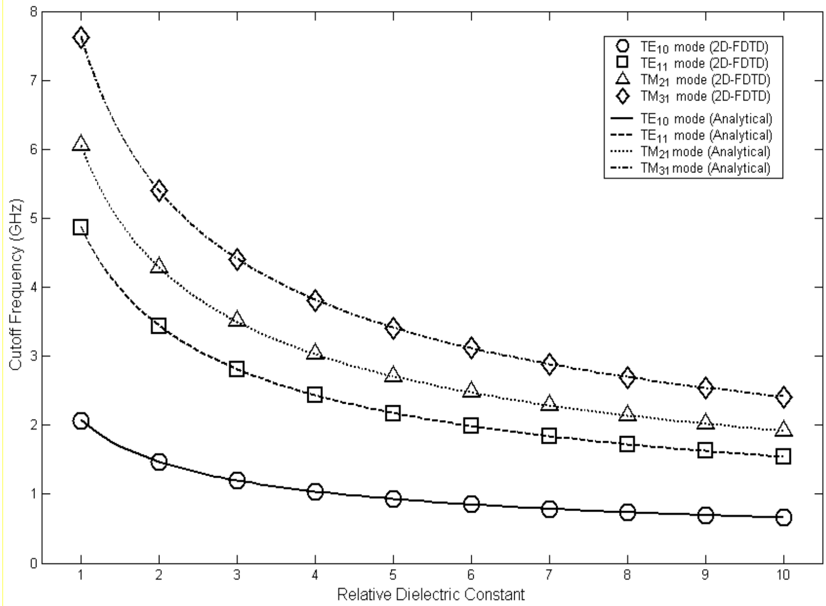

Fig. 10. Variation of mode cut-off frequency versus different relative permittivity.

5 CONCLUSION

In this paper, 2-D FDTD method has been introduced to analyze widely used rectangular waveguides. The present algorithm is validated by comparing the computational results with the exact solutions. It is believed that a technique which is accurate, yet retains the interactive design capabilities of the simpler techniques has been developed and its accuracy has been shown by comparing with analytical results.

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