New Compensation Functions for Efficient Excitation of Open Planar Circuits in SDM

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Abstract—The spectral-domain method is a very fast and powerful technique to analyze planar microwave circuits. However, available techniques for simulating the excitation of open planar microwave circuits are not very effective at relatively low frequencies. In this paper, new compensation functions are introduced, which correctly model the excitation over the whole microwave frequency region.

Index Terms—Excitation, planar circuits, SDM.

I. INTRODUCTION

This paper highlights the lack of information and drawbacks of available excitation modeling techniques for the derivation of the S-parameters of open planar circuits and presents a novel compensation function for efficient excitation modeling.

The details available in the literature for the extraction of the S-parameters of planar circuits using the spectral-domain method (SDM) or other related techniques are limited in scope, as the topic is avoided or ignored in most papers. For shielded planar microwave circuits, the S-parameters at a spot frequency are derived in [1, Ch. 5] from the solution of a matrix equation. In [1], the tangential electric field is assumed to be identically zero because of the sidewalls. Since they do not exist in open structures, a different method must be sought. In addition, the effects of the port/ feed arrangement are assumed to be negligible. The extraction of the S-parameters from the knowledge of the surface current distribution is achieved by applying transmission-line theory to the feedlines (deembedding algorithm) [2], but this method can only be used when the line length is bigger than a half-wavelength. A method was introduced by Jackson [3] to calculate S-parameters of gap discontinuities in 1985 for open planar circuits, but his technique, in contrast to the method described here, is not complete and efficient at relatively low frequencies. This is because the cosine portion of the traveling wave is truncated one-quarter of a guide wavelength from a zero of the sine. The length of the truncated portion is a function of the operating frequency. At low frequencies, its length becomes larger than the entire circuit’s dimension and a large number of extra rooftop functions are required in order to avoid spurious numerical reflections.

In this paper, semi-infinite feedlines, which extend to infinity from the ports, are connected to the ports of the circuit to simplify the excitation. However, in order to complete the algorithm, an additional set of basis functions is required to define the unknown current distribution on the feedlines. Following [4], traveling current waves are used as basis functions. With this choice, the S-parameters of the circuit can be directly derived from the coefficients of the traveling waves used. However, the use of the traveling wave as a current basis function on the feedline causes a current discontinuity in the interface between the port and feedline. This difficulty was overcome in [3] and [4] by truncating the cosine (real part) portion a quarter-wavelength from a zero of the sine (imaginary part). However, this truncation requires an extra number of rooftop functions, which is a large number at low frequencies. Here, in order to overcome this difficulty and to reach an accurate solution, a function, which is named by the authors as the compensation function, is introduced at the interface between the port of the circuit and adjacent feedline. The accuracy and efficiency of the new method is demonstrated by means of numerical examples.

II. IMPROVED EXCITATION MODELING TECHNIQUE

For the formulation described in this paper, all circuits are considered to be connected to a semi-infinite feedline, whose width is identical to the joining port, at each port. The fundamental microstrip mode is assumed to propagate on the feedlines and, thus, the traveling current waves are chosen as current basis functions given in

\[ J_i(z) = \begin{cases} e^{-j k_n(z-z_i)}, & -L + z_i \leq z \leq z_i \\ 0, & \text{otherwise} \end{cases} \]

\[ J_s(z) = \begin{cases} -a_s e^{j k_n(z-z_i)}, & -L + z_i \leq z \leq z_i \\ 0, & \text{otherwise} \end{cases} \]

\[ J_o(z) = \begin{cases} a_o e^{-j k_n(z-z_i)}, & z_o \leq z \leq z_i + L_o \\ 0, & \text{otherwise} \end{cases} \] (1)

Here, \( k_n \) is the precalculated wavenumber of the feedline, \( L \) is the length of the feedline, which theoretically extends to infinity, but in practice, is chosen to be an integer number of half-wavelengths [3], [4], and \( z_s, z_i, z_o \) is the offset of the port from the origin. The letters \( i, t, r \) indicate the incident, transmitted, and reflected current waves, respectively, and the unknown coefficients \( a_s, a_o \) are coefficients of the reflected and transmitted current waves which are to be calculated. For the excited and nonexcited ports, the currents are

\[ J_{\text{input}} = J_i + J_r \]

\[ J_{\text{output}} = J_t \] (2)

As shown in Fig. 1, the current basis functions of the feedlines have a real \((\cos k_n z)\) and an imaginary \((\sin k_n z)\) part. The real part causes current discontinuities in the direction of current flow because the triangle function as a rooftop function’s component is identically zero at the interface, whereas \(\cos(k_n z)\) has a finite value.

Similarly, in the direction perpendicular to current flow, the step function as a rooftop function’s component has a finite value, whereas \(\sin(k_n z)\) is identically zero. Therefore, extra basis functions are still required at each interface between the port and adjacent feedline for the correct solution. These functions are introduced and referred to by the authors as compensation functions, which are shown in Fig. 2. The compensation functions are defined for the input port in the space domain by

\[ J_i(z) = \begin{cases} 1 - \frac{\sin z_i}{z}, & z_i \leq z \leq z_i + L_i \\ 0, & \text{otherwise} \end{cases} \] (3)

\[ J_s(z) = \begin{cases} \frac{\sin z}{z}, & z_i \leq z \leq z_i + L_i \\ 0, & \text{otherwise} \end{cases} \] (4)
and for the output port

\[ J_1(z) = \begin{cases} \frac{z - z_o}{l_1}, & z_o \leq z \leq z_o \\ 0, & \text{otherwise} \end{cases} \]  \hspace{1cm} \text{(5)}

\[ J_2(z) = \begin{cases} \frac{z - z_o}{l_2}, & z_o \leq z \leq z_o \\ 0, & \text{otherwise} \end{cases} \]  \hspace{1cm} \text{(6)}

where \( l_1 \) is the size of the compensation functions in the direction of propagation.

\( J_1(z) \) in (3) and Fig. 2(a) transfers the effect of the cosine portion of the incident current wave in the direction of current flow, whereas \( J_2(z) \) in (4) and Fig. 2(a) transfers the effect of the sine portion of the incident current wave in the direction perpendicular to the current flow. Similarly, \( J_1(z) \) in (5) and Fig. 2(b) transfers the current wave to the output port in the direction of propagation, whereas \( J_2(z) \) in (6) and Fig. 2(b) transfers the current wave in the transverse direction. Including all current basis functions of the entire system, the total current is expressed as

\[ \mathbf{J}_\text{total} = \mathbf{J}_\text{inc} + \sum_{n=1}^{N} (\mathbf{a}_{p,n} \mathbf{J}_\text{port}_{n}(k_x, k_y)) + \mathbf{a}_{e} \mathbf{J}_\text{cosine}(k_x, k_y) \]  \hspace{1cm} \text{(7)}

where \( N \) is the number of ports, \( \mathbf{J}_\text{inc} \) refers to the basis functions of the microwave circuit, \( \mathbf{J}_\text{port}_{n} \) is either the sum of the incident and reflected current wave for the excited port or the transmitted current wave for unexcited ports, and \( \mathbf{J}_\text{cosine} \) is the Fourier transform of the compensation function. To calculate the \( S \)-parameters of the circuit, \( \mathbf{a}_{p,n} \) must be known. The method of moments is employed to find the unknown coefficients. A total of \( N \) weighting functions must be defined to complete the algorithm. These are chosen to be a triangle function, which straddles the lines separating each port and the feedline in the direction of current flow and a precalculated basis function in the direction perpendicular to current flow. After application of the method of moments, the matrix equation yields

\[ [\mathbf{Z}]_{n} [\mathbf{I}]_{n} = [\mathbf{Z}], \]  \hspace{1cm} \text{(8)}

where \( \mathbf{Z} \) consists of the elements related to unknown current basis functions and \( \mathbf{Z} \) is a column vector containing the elements related to incident current waves. The unknown coefficients are found by solving (9). The \( S \)-parameters of the \( N \)-port circuit are actually the coefficients of the reflected and transmitted current waves.

III. NUMERICAL RESULTS

It must be noted that the Jackson excitation technique is used with no extra rooftop functions to complete the truncated portion of the current wave. This is due to using an identical number of basis functions in each model and illustrating the improvement in the accuracy for an identical number of basis functions.

A. Microstrip Line

The microstrip line, which is of length 2.8 mm, width 0.7 mm on the substrate of thickness 0.381 mm, relative permittivity 11.7, and relative permeability 1, has been chosen. The reason for this choice is that the geometry is very simple, its \( S \)-parameters are very easy to predict. The dimensions of the circuit have been intentionally chosen to be relatively small in order to highlight the deficiency of Jackson’s technique and to illustrate the improvement in the accuracy which can be obtained.

Four sets of \( S \)-parameter results are plotted in Fig. 3. These are the results calculated by using 42 rooftop functions in total, having dimensions of \( d = w/\lambda = 0.157 \) mm and \( l_1 = L/8 = 0.35 \) mm. First, the test structure is analyzed by using truncated traveling current waves for excitation. The minimum operating frequency in the analysis is 1 GHz and the corresponding length of the truncated cosine portion is 26.25 mm, which is 75 times bigger than the size of the overlapping rooftop. For the maximum operating frequency of 20 GHz, the corresponding length of the truncated portion is 1.25 mm, which is still more than three times larger than the rooftop. Therefore, the cosine portion of the incident current wave never reaches the input port of the circuit in this frequency range and the circuit behaves like an open-end discontinuity, as shown in Fig. 3. To get the accurate results by Jackson’s technique for this frequency range, 888 extra rooftop functions are required.

Finally, the introduced excitation modeling is used for the analysis for the test structure, and the improvement in the accuracy is highlighted in Fig. 3 by using a compensation function for each port instead of 888 extra rooftop functions. In this model, the effects of
excitation are transferred into the circuit without any loss. The $S_{21}$ is almost unity, whereas $S_{11}$ is very small, as would be expected.

### B. Edge-Coupled Filter

In order to further prove the accuracy of the excitation modeling introduced here, using a more complicated structure, the analysis of the microstrip edge-coupled filter shown in [5, Fig. 6] is considered. The measurements performed by Shibata et al. [6] are used as a standard for comparison.

As seen in Fig. 4, there is a clear agreement between the introduced excitation modeling and measured data. The use of traveling current waves with truncation yields completely meaningless results because the minimum operation frequency is 1 GHz with the corresponding length of the truncated cosine portion being 29.14 mm. This is almost 46 times bigger than the size of the rooftop in the direction of the propagation for the definition ($l_s = w/4 = 0.318$ mm and $l_c = l/20 = 0.636$ mm). Even at the maximum operating frequency, which is 15 GHz, extra rooftops are still required to cover the missing portion. For the maximum operating frequency, the length of the truncated rooftop is 1.78 mm and still three times larger than the rooftop size. Again, for the same accuracy as for the Jackson technique, 540 extra rooftops (270z and 270z components) are required to complete the missing portions.

### IV. Conclusion

We have shown that realistically complex microstrip circuits can be rigorously analyzed at relatively low frequencies using new compensation functions, which are defined at the interface between a port and feedline in the SDM. The compensation functions are shown to be effective to model the effect of the excitation of the circuit.

### References


### Direct Measurement of $C_{1e}$ and $C_{1b}$ Versus Voltage for Small HBT’s with Microwave $s$-Parameters for Scaled Gummel–Poon BJT Models

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**Abstract—** A novel method for determining the junction capacitances versus voltage in heterojunction bipolar transistors (HBT’s) using $s$-parameters at microwave frequencies is presented. This new technique has several advantages over traditional approaches, which include: 1) it profiles capacitance at greater forward bias; 2) it enables the direct measurement of minimum geometry transistors; 3) it allows for the accurate extraction of scaled HBT model parameters with emitter length; and 4) it results in improved pad parasitic deembedding for accurate modeling. Both the capacitance–voltage and large-signal HBT model results are shown.

**I. Introduction**

The accurate determination of the heterojunction bipolar transistor (HBT) base–emitter (BE) and base–collector (BC) junction capacitance with voltage is critical for large-signal device modeling. Traditional approaches rely on capacitance–voltage (CV) meters, which are limited by the leakage current (which sets maximum forward bias) and low frequency of operation (which sets minimum measurable capacitance). The first limit prevents measurement in the voltage regime where $C_{b,e}$ and $C_{b,c}$ change significantly (before the diffusion capacitance dominated region). In high-speed HBT’s at low bias, $C_{b,e,junction}$ may dominate $C_{b,e,diffusion}$; thus, errors in extrapolation $C_{b,e}$ from low $V_{b,e}$ measurements can result in large errors in predicting the high-speed behavior. In the devices reported here, $C_{b,e,junction} > C_{b,e,diffusion}$ for $I_c < 0.2$ mA. The second limit requires large test-pattern measurements which are then scaled to the device; thus, resulting in errors for small HBT’s due to periphery effects and fabrication tolerances. The proposed CV technique based on microwave (1–50 GHz) s-parameters (RF–CV) increases the tolerable forward bias range and allows direct measurement of devices.