# EEE321 Electromagnetic Fields and Waves

Prof. Dr. Hasan Hüseyin BALIK

($8^{th}$ Week)

# Outline

- Lorentz Force
- Magnetic Force on a Current-Carrying Wire
- Magnetic Force between two Closed Circuit
- Magnetic Force on a Volume Distribution of Current

# The Lorentz Force

- Experiments Show that the force exerted on a particle of charge *Q* moving ina vacuum at an instantaneous velocity ***v*** in a region where exist both an electric field and magnetic field is

$$\boldsymbol{F} = Q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}). \qquad (22\text{-}1)$$

- This equation is valid even if ***v*** approaches the speed of ligth. **E, B** and ***v*** can be time dependent but all concern the same reference frame
- Magnetic force ($Q\boldsymbol{v}x\boldsymbol{B}$) is magnetic force and is perpendicular to ***v***. Therefore
  - it can change the direciton of ***v***
  - it can not alter its magnetute
  - İt can not alter the kinetic energy

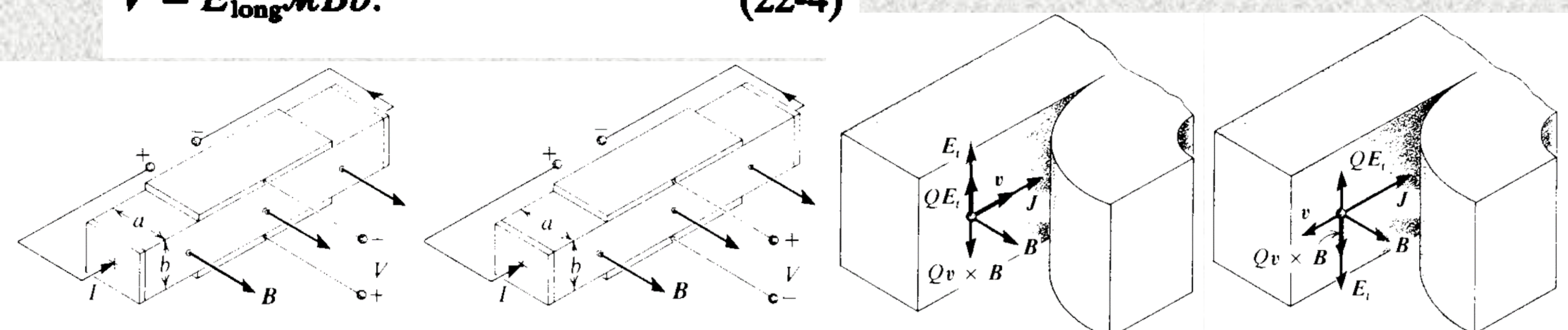
# Example: The Hall Effect

- In a bar of conducting material subjected to electri and magnetic field as in figure (a) and (b)
- They also drift sideways because of $Q\boldsymbol{v}x\boldsymbol{B}$ force
- As a result there appears a potantial difference between the upper and lower electrodes
- This is called the **Hall Effect**

$$QE_t = \frac{QV}{b} = QvB, \qquad (22\text{-}2)$$

$$V = vBb. \qquad (22\text{-}3)$$

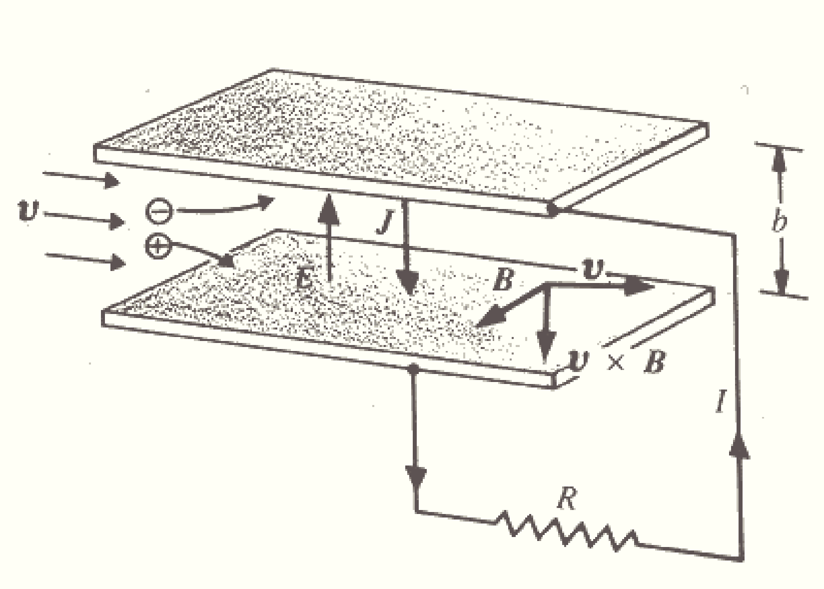
$$V = E_{\text{long}}\mathcal{M}Bb. \qquad (22\text{-}4)$$




(a) (b) (c) (d)

# Example: Magnetohydrodynamic Generator

- Magnetohydrodynamic (MHD) generator is a large scale application of the hall effect.
- It converts part of the kinetic energy of a hot gas direcly into electric energy
- A hot gas enters on the left at a velocity of the order of 1000 m/s
- It contains salt such as ($K_2CO_3$) that ionizes readily at high temp.
- The temp. approaches 3000 $K^0$
- Conductivity is about 100 S/m




- Positive ions curve downward and electrons upward
- The resulting current *I* flows through the load resistance *R*
- The advantage of MHD generator is that comprises no moving parts except for the gas

- The function of magnetic force is to compel the positive particules to go to the positive electrode and the negative particules to go to negative electrode.
- Under the action of the forces shown in figüre, both move uphill and slow down.
- The Lorentz force acts on a charge *Q* as if the electric filed strength were $\mathbf{E} + Q\boldsymbol{v}x\boldsymbol{B}$. So for the gaz conductivity σ,

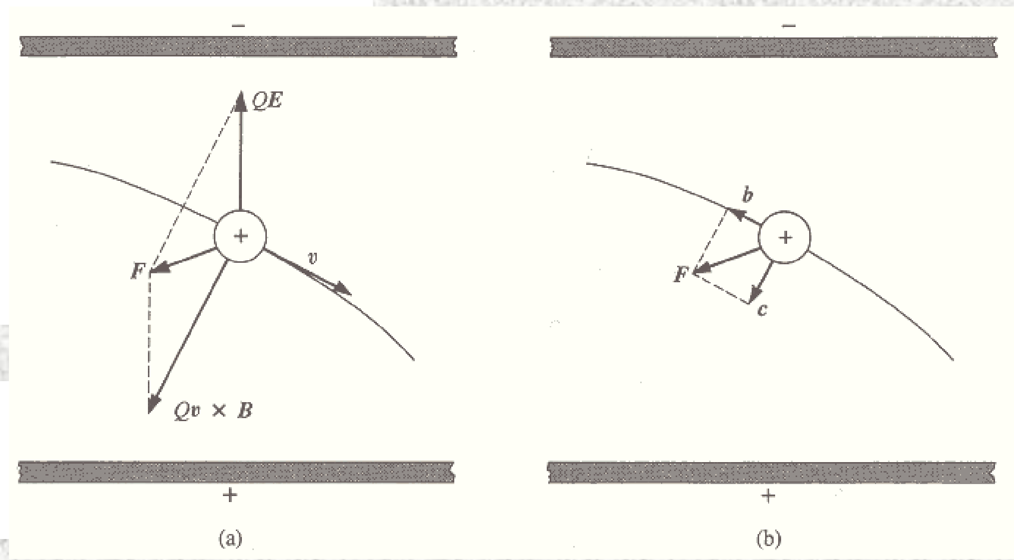
$$|\boldsymbol{J}| = |\sigma(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})| = \sigma(vB - E) = \sigma\left(vB - \frac{V}{b}\right), \quad (22\text{-}6)$$

$$I = \mathcal{A}J = \mathcal{A}\sigma\left(vB - \frac{IR}{b}\right),$$

$$I = \frac{vBb}{b/(\sigma\mathcal{A}) + R}.$$

- The outout voltage is

$$V = IR = vBb - \frac{Ib}{\sigma\mathcal{A}}.$$



- Let us define the efficieny as

$$\mathcal{E} = \frac{\text{Joule losses in } R}{\text{Joule losses in } R + \text{Joule losses in the gas}} \qquad (22\text{-}10)$$

$$= \frac{I^2R}{I^2R + I^2b/(\sigma\mathcal{A})} = \frac{R}{R + b/(\sigma\mathcal{A})}, \qquad (22\text{-}11)$$

- Form the above expression for *I* and for *IR*,

$$\mathcal{E} = R\frac{I}{vBb} = \frac{IR}{vBb} = 1 - \frac{I}{\sigma\mathcal{A}vB}. \qquad (22\text{-}12)$$

- The efficiency is equal to unity when $I=0$ or when $R \to \infty$.
- The efficiency is equaş to zero when $I=\sigma AvB$ or $R=0$, $V=0$

# Magnetic Force on a Current Carrying Wire

- A stationary wire of cross section *A* carries a current *I* in a region where there exist a megnetic field **B** originating elsevere
- The wire contains *N* charge carriers per cubic meter drifting at a velocity ***v***, each one of charge *Q*.
- An element of length ***dl*** of a wire contains *ANdl* charge carriers. Then the magnetic force on ***dl*** is

$$d\mathbf{F} = \mathscr{A}N\,dl\,Q\mathbf{v} \times \mathbf{B} = (\mathscr{A}NQv)\,d\mathbf{l} \times \mathbf{B} = I\,d\mathbf{l} \times \mathbf{B}, \qquad (22\text{-}13)$$

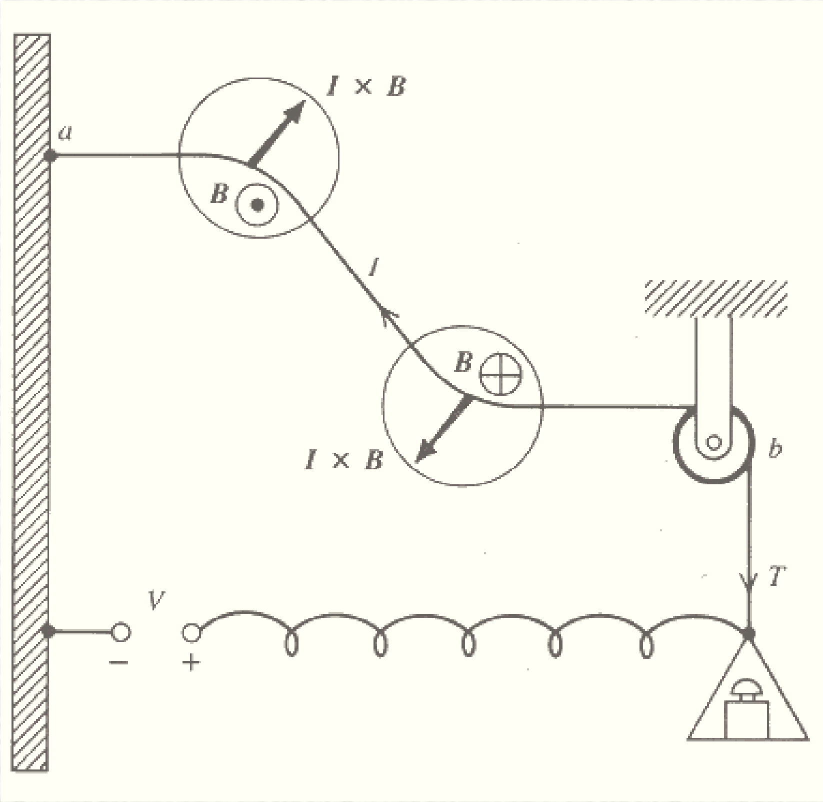
- The magnetic force per unit length is therefore ***IxB***
- The total magnetic force on a closed circuit *C* carriying a current *I* and lying in a magnetic field is

$$\mathbf{F} = I \oint_C d\mathbf{l} \times \mathbf{B}. \qquad (22\text{-}14)$$

# Example: Floating-Wire Hodoscope

- The floating-wire hodoscope is a device that simulates the trajectory of a charges particle in a magnetic field.
- Say a charged particle of mass *m*, charge *Q* and velocity ***v*** follows a certain trajectory in going from a point *a* to a point *b* in a magnetic field.
- A ligth wire carring a current *I*, fixed at *a* and going over a pulley at *b,* in the same magnetic field, as in the figüre, will follow that trajectory if

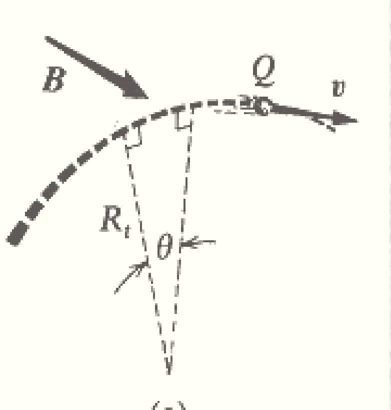
$$\frac{mv}{Q} = \frac{T}{I}, \qquad (22\text{-}15)$$




- *T* is the tension in the wire

- Suppose the beam is normal to **B** as in fig. (a)
- $R_t$ is the radius of curvature of the trajectory

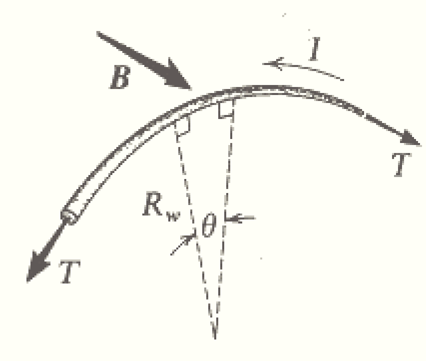
$$QvB = \frac{mv^2}{R_t}, \qquad R_t = \frac{mv}{QB}. \qquad (22\text{-}16)$$



(a)

- Suppose the wire is also normal to **B** as in fig(b)
- The element *dl*, with a radius of curvature $R_w$ is in equilibrium if the outward magnetic force *BIdl* just compensates the inward component of the tension force *T:*

$$BI\,dl = 2T\sin\frac{d\theta}{2} = T\frac{dl}{R_w}, \qquad R_w = \frac{T}{IB}. \qquad (22\text{-}17)$$



(b)

- The two radii are equal if $\frac{mv}{Q} = \frac{T}{I}$. (22-18)

# Magnetic Force between two Closed Circuits

- The magnetc force exerted by a current $I_a$ on a current $I_b$ is given by

$$\mathbf{F}_{ab} = I_b \oint_b d\mathbf{l}_b \times \frac{\mu_0}{4\pi} I_a \oint_a \frac{d\mathbf{l}_a \times \hat{\mathbf{r}}}{r^2},$$

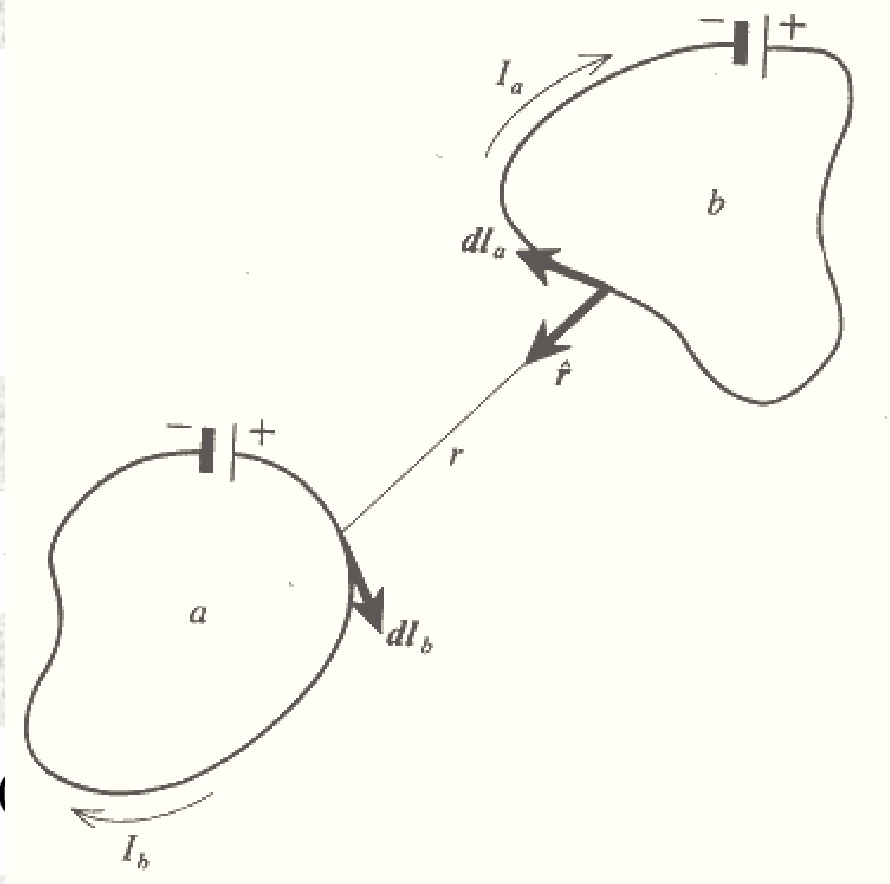
$$= \frac{\mu_0}{4\pi} I_a I_b \oint_a \oint_b d\mathbf{l}_b \times \frac{d\mathbf{l}_a \times \hat{\mathbf{r}}}{r^2},$$

- To Show that $\mathbf{F}_{ab} = -\mathbf{F}_{ba}$, first

$$d\mathbf{l}_b \times (d\mathbf{l}_a \times \hat{\mathbf{r}}) = d\mathbf{l}_a (d\mathbf{l}_b \cdot \hat{\mathbf{r}}) - \hat{\mathbf{r}} (d\mathbf{l}_a \cdot d\mathbf{l}_b)$$

- Then, rearranging terms,

$$\oint_a \oint_b \frac{d\mathbf{l}_a (d\mathbf{l}_b \cdot \hat{\mathbf{r}})}{r^2} = \oint_a d\mathbf{l}_a \oint_b \frac{d\mathbf{l}_b \cdot \hat{\mathbf{r}}}{r^2}.$$




- This integral is zero because *b* is closed and

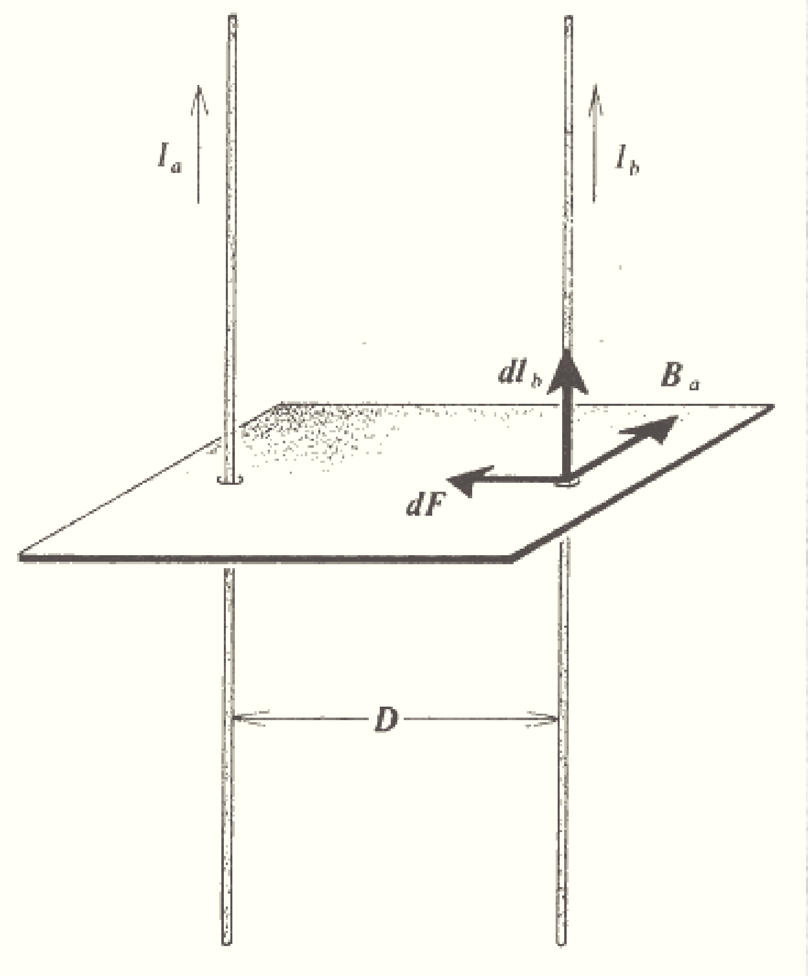
$$\mathbf{F}_{ab} = -\frac{\mu_0}{4\pi} I_a I_b \oint_a \oint_b \hat{\mathbf{r}} \frac{d\mathbf{l}_a \cdot d\mathbf{l}_b}{r^2}. \qquad (22\text{-}23)$$

# Force between two paralel Currents

- Without integration, we can calculate the force per unit length between two long paralel wires bearing current
- At the position of $I_b$, $B_a$ is $\mu_o I_a/2\Pi D$ in the direction shown in the figüre
- The force on a unit length of wire *b* is thus

$$F' = B_a I_b = \frac{\mu_0 I_a I_b}{2\pi D}. \qquad (22\text{-}24)$$

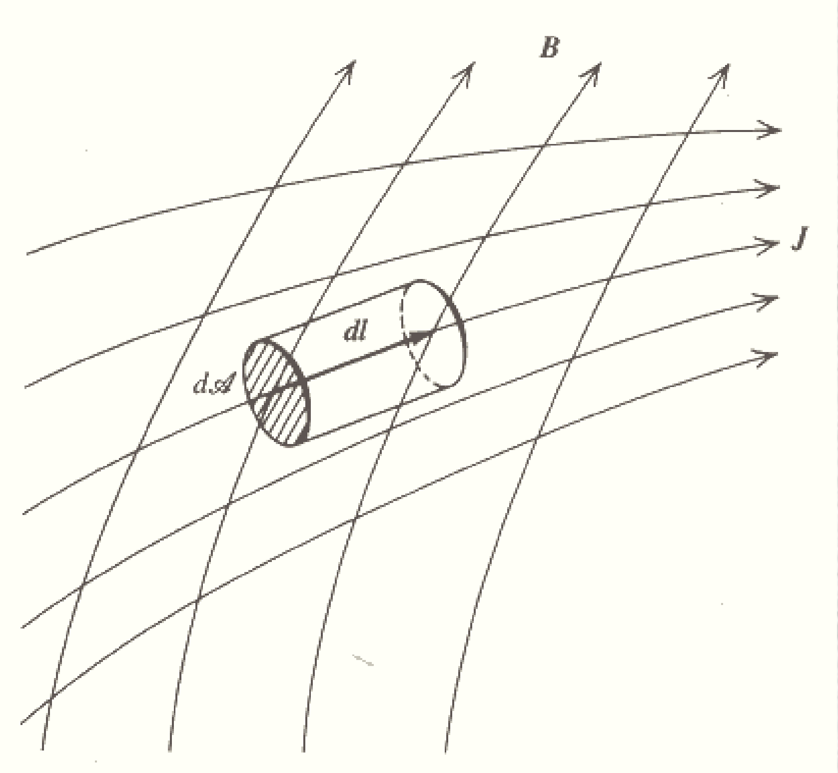
- The force is attractive if the current flow in the same direction and repulsive otherwise.
- This force is normaly negligible



## Magnetic Force on a volume Distribution of Current

- Consider a small element of volüme of length ***dl*** paralel to ***J*** and of cross section of *dA,* as in figure.
- The magnetic force on the element is

$$\mathbf{dF} = (J\, d\mathscr{A})\, \mathbf{dl} \times \mathbf{B} = \mathbf{J} \times \mathbf{B}\, dv$$




- The force per unit volume is

$$\mathbf{F}' = \mathbf{J} \times \mathbf{B}. \qquad (22\text{-}30)$$

- The total magnetic force on a given distribution of conductiing currents occupying a volüme *v* is

$$\mathbf{F} = \int_v \mathbf{J} \times \mathbf{B}\, dv. \qquad (22\text{-}31)$$