EEE321 Electromagnetic Fields and Waves

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(6th Week)

Outline

- Type of Magnetic Materials
- Magnetizasyon **M**
- Magnetic field Strength **H** and Curl of **H**
- Dielectric and Magnetic Materials Compared
- Magnetic Susceptibility and Relative Permeability
- Boundary Conditions

Type of Magnetic Material

- (1) All materials are **diamagnetic.** The application of external magnetic field induces moments according to the Faraday induction law. This effect is usualy inperceptable and it disappears upon removal of the external field.
- (2) In ost atoms, the magnetic moments resulting from orbitral and spinning motion of the electrons cancel. If the calcellation is not complate, the material is **paramagnetic.**
- (3) In some materials, the magnetizasyon can be orders of magnitiute larger than in either diamagnetic nor paramagnetic substance. This material is **ferromagnetic.**

Magnetization M

- The **magnetization M** is the magnetic moment per unit volüme of magnetized material at a point.
- If there are *N* atoms per unit volüme, each possessinga magnetic dipol moment **m** oriented in a given direction, then

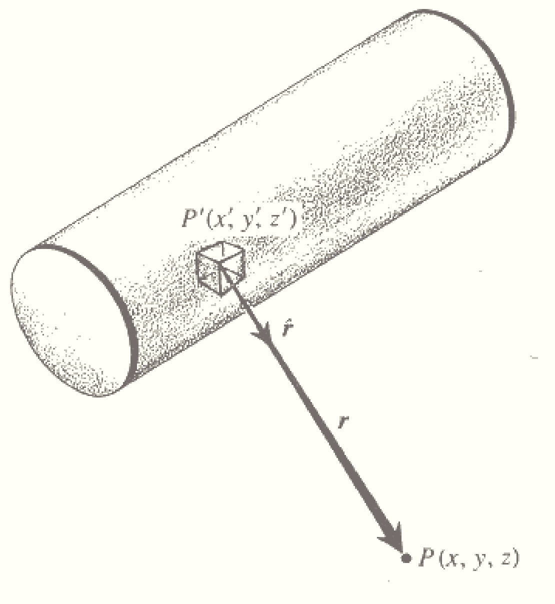
$$\boldsymbol{M} = N\boldsymbol{m}. \qquad (20\text{-}1)$$

- Themagnetization **M** in magnetic madia corresponds the polarization **P** in dielectric.
- The unit of magnetization is ampere per meter

Magnetic Field of A Magnetized Bady

- Let us calculate **B** at a point outside a magnetized bady, as in figure.
- The vector potential at a point *P* located at a distance *r* from a current loop of magnetic moment **m** is

$$\mathbf{A} = \frac{\mu_0}{4\pi} \frac{\boldsymbol{m} \times \hat{\boldsymbol{r}}}{r^2}. \tag{20-2}$$



- Then, for a volüme *v′* of magnetized material,

$$\mathbf{A} = \frac{\mu_0}{4\pi} \int_{v'} \frac{\boldsymbol{M} \times \hat{\boldsymbol{r}}}{r^2} dv' = \frac{\mu_0}{4\pi} \int_{v'} \boldsymbol{M} \times \nabla'\left(\frac{1}{r}\right) dv'. \tag{20-3}$$

$$\mathbf{A} = -\frac{\mu_0}{4\pi} \int_{v'} \left(\nabla' \times \frac{\boldsymbol{M}}{r}\right) dv' + \frac{\mu_0}{4\pi} \int_{v'} \frac{\nabla' \times \boldsymbol{M}}{r} dv' \tag{20-4}$$

$$= \frac{\mu_0}{4\pi} \int_{\mathcal{A}'} \frac{\boldsymbol{M} \times \hat{\boldsymbol{n}}}{r} d\mathcal{A}' + \frac{\mu_0}{4\pi} \int_{v'} \frac{\nabla \times \boldsymbol{M}}{r} dv', \tag{20-5}$$

- It is clear that the vector potential in the neighborhood of a piece of magnetized materila is the same as if one had volume and surface current densities. These equivalent current s are;

$$\boldsymbol{J}_e = \nabla \times \boldsymbol{M} \qquad \text{and} \qquad \boldsymbol{\alpha}_e = \boldsymbol{M} \times \hat{\boldsymbol{n}}. \qquad (20\text{-}6)$$

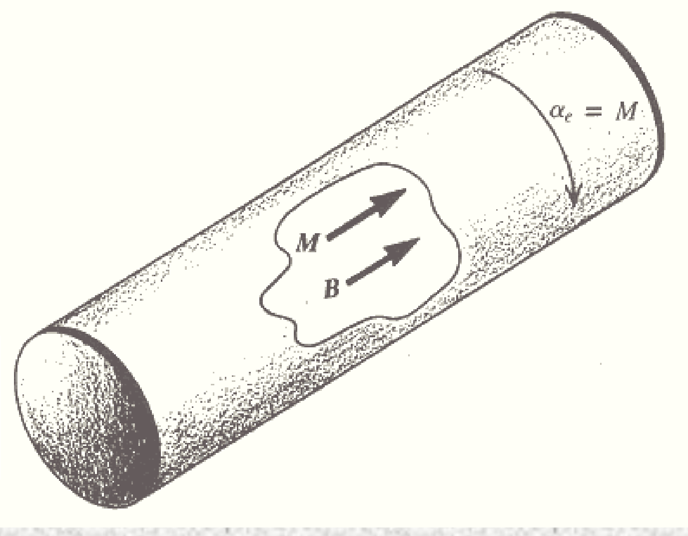
- More generaly;

$$\mathbf{A} = \frac{\mu_0}{4\pi} \int_{v'} \left(\boldsymbol{J}_f + \frac{\partial \boldsymbol{P}}{\partial t} + \nabla \times \boldsymbol{M} \right) dv', \qquad (20\text{-}7)$$

- Thus a more general form of the Biot-Savart law

$$\boldsymbol{B} = \frac{\mu_0}{4\pi} \int_{v'} \frac{(\boldsymbol{J}_f + \partial \boldsymbol{P}/\partial t + \nabla \times \boldsymbol{M}) \times \hat{\boldsymbol{r}}}{r^2} dv'. \qquad (20\text{-}8)$$

Divergence of B in Presence of Magnetic Material



- The magnetif fields originate either in the macroscopic motion of charge or in equivalent current.
- The relation

$$\nabla \cdot \boldsymbol{B} = 0 \qquad (20\text{-}9)$$

applies even in the presence of magnetic materials.
- This is one of Maxvell's equations.

The Magnetic Field Strength H and $\nabla x H$

- For the static fields in the absence of magnetic materials

$$\nabla \times B = \mu_0 J_f. \tag{20-10}$$

- J_e is the current density related to the motion of free charges.
- In the presence of magnetized materials,

$$\nabla \times B = \mu_0 (J_f + J_e). \tag{20-11}$$

$$\nabla \times B = \mu_0 (J_f + \nabla \times M), \tag{20-12}$$

$$\nabla \times \left(\frac{B}{\mu_0} - M \right) = J_f. \tag{20-13}$$

- The vector within the parantesis called **magnetic field strength**

$$H = \frac{B}{\mu_0} - M. \tag{20-14}$$

- Both **H** and **M** are expressed in amperes / meter. Thus

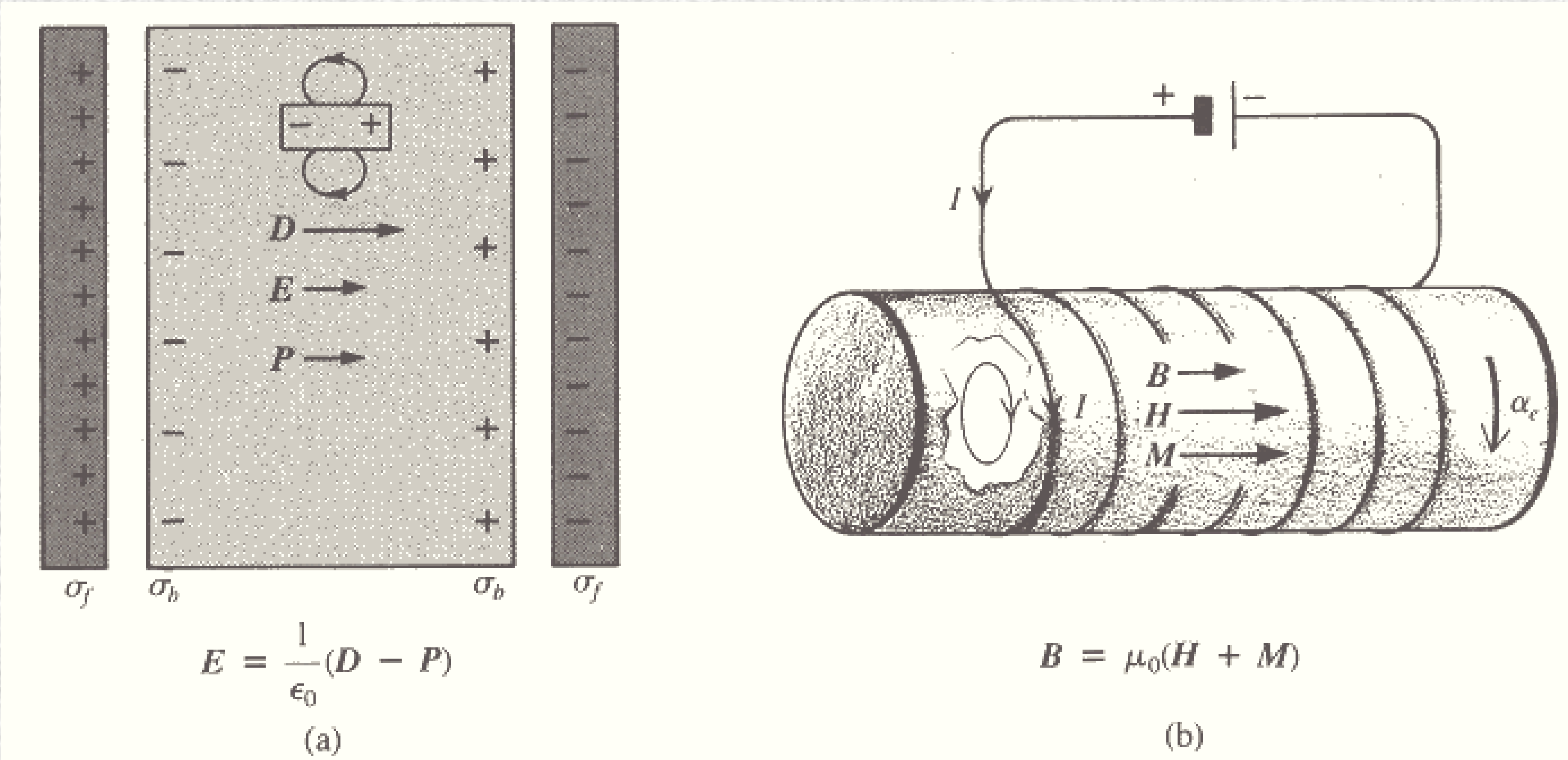
$$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M}) \qquad (20\text{-}15)$$

- And even inside the magnetized material,

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}_f \qquad (20\text{-}16)$$

- It must be rememberred that above equation is only valid in static fields

Dielectric and Magnetic Material Compared



- In dielectric E is smoller because the field of the bound charges opposes that of the free charges
- In magnetic material, **B** is larger because the field of the equivalent current aids that of the free current

Ampere's Circuital Law in Presence of Magnetic Material

- Let as integrate below equation over an open surface of area A bounded by a curve C

$$\nabla \times \mathbf{H} = \mathbf{J}_f \qquad (20\text{-}16)$$

- We found:

$$\int_{\mathcal{A}} (\nabla \times \mathbf{H}) \cdot d\mathcal{A} = \int_{\mathcal{A}} \mathbf{J}_f \cdot d\mathcal{A}, \qquad (20\text{-}18)$$

- Using Skotes theorem on the left-hand side,

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = I_f, \qquad (20\text{-}19)$$

- Note that I_f does not include the equivalent currents. It can serve to calculate **H** even in the presence of magnetic materials. This is valid fpr only steady currents

Magnetic Susceptibility *(X_e)* and Relative Permeability μ_r

- It is convenient to define magnetic susceptibility X_m such that

$$M = \chi_m H. \tag{20-21}$$

- Then

$$B = \mu_0(H + M) = \mu_0(1 + \chi_m)H = \mu_0\mu_r H = \mu H, \tag{20-22}$$

- where

$$\mu_r = 1 + \chi_m \tag{20-23}$$

- is the relative permeability, and

$$\mu = \mu_r\mu_0 \tag{20-24}$$

- is the permeability of a material. Thus

$$M = \chi_m \frac{B}{\mu}. \tag{20-25}$$

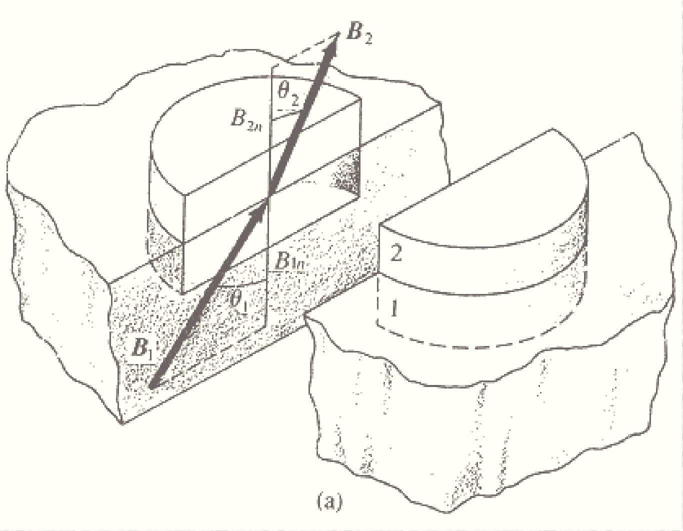
Boundary Condition

- Both **B** and **H** obey boundary conditions at the interface between two media
- The net outward flux of **B** through any closed surface is zero

$$\oint_{\mathcal{A}} \mathbf{B} \cdot d\mathcal{A} = 0. \qquad (18\text{-}18)$$

- Then the normal component of **B** is continuous accross an interface

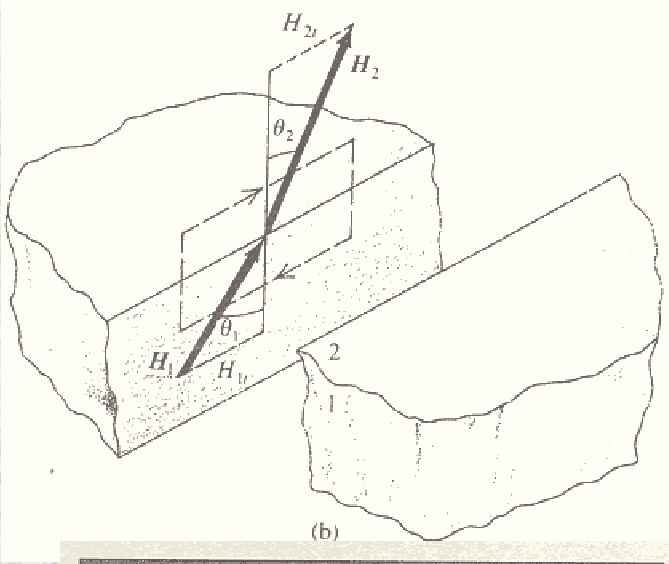
$$B_{1n} = B_{2n}. \qquad (20\text{-}26)$$



(a)

- Form the circuital law, the line integral of **h.dl** around a path is equal to the current *I* linking the path

$$\oint_C \boldsymbol{H} \cdot d\boldsymbol{l} = I_f, \qquad (20\text{-}19)$$



- At the interface, if the current is zero then the tangential component of **H** is continuous across the interface

$$H_{1t} = H_{2t}. \qquad (20\text{-}27)$$

- Setting **B=μH** for both media, Then

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_{r1}}{\mu_{r2}}. \qquad (20\text{-}28)$$