EEE321 Electromagnetic Fileds and Waves

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(4th Week)

Outline

- Basic Polarization Processes
- The Electric Plarization
- Free and Bounded Charges
- The Electric Field of a Polarized Dielectric
- Gauss’s Law
- The Electric Flux Density
- Continuity Conditions at an Interface
- The Potential Energy in the Presence of Dielectric

Three Basic Polarization Processes

- (1) Under the action of an applied electric field, the center of charge of the electron cloud moves with respect to the center of the nuclei. This is called **electronic polarization**
- (2) Polar molecules aling themselves and become futher polarized in an applied electric field. This is called **orientational polarization**
- (3) ion of different sign in a solid such as NaCl move in different directions when subjected to an electric field. This is called **atomic polarization**

Electric Polarization (P)

- If the neighbourhood of a given point, the average vector dipole moment per molecule in a given direcion is **p**, and if *N* molecules per cubic meter, then

$$P = Np \qquad (9\text{-}1)$$

  is the **electric polarization** at that point.

- So **P** is defined as the **dipole moment per unit volume**

Free and Bound Charges

- Polarization causes charges to accumulate either within the dielectric or its surface. This charges called **bound.**
- The conduction electrons in good conductors are said to be **free**

Bound Surface Charge Density (σ_b)

- Imagine an element of area *dA* inside a nonpolar dielectric as in Figure. When the dielectric is polarized, the center of positive charge +Q of a molecule lies at a distance *s* from the center of negative –Q.
- Upon application of an electric field, n_+ positive charges cross the element of area by moving, n_- negative charges cross it by moving oposite direction. The net charge that crosses *dA* is

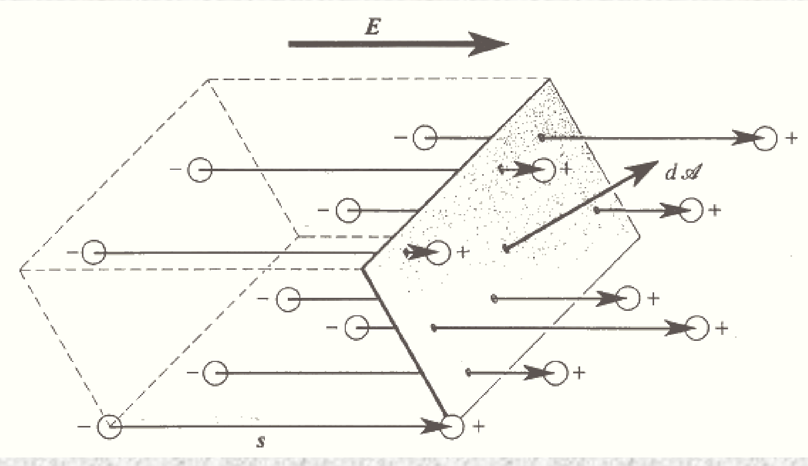
$$dQ = n_+Q - n_-(-Q) = (n_+ + n_-)Q. \quad (9\text{-}2)$$

- Where n_+ and n_- is the umbe rof molecules within the imaginary **s.dA** volüme

$$dQ = NQ\mathbf{s} \cdot d\mathcal{A} = N\mathbf{p} \cdot d\mathcal{A} = \mathbf{P} \cdot d\mathcal{A}, \quad (9\text{-}3)$$

- Where Q**s** is the dipole moment **p**
- The bound surface charge density

$$\sigma_b = \frac{dQ}{d\mathcal{A}} = \mathbf{P} \cdot \hat{\mathbf{n}}, \quad (9\text{-}4)$$



Bound Volume Charge Density (ρ_b)

- The bound charge that flows out of the closed surface of area *A* delimiting a volume *v* entirely situated within the dielectric is

$$Q_{out} = \int_{\mathcal{A}} \mathbf{P} \cdot d\mathcal{A}, \qquad (9\text{-}5)$$

- Net charge that remains within *v* must be –Qout. If ρ_b is the volume density of the charge remaining within *v* is

$$\int_v \rho_b \, dv = -Q_{out} = -\int_{\mathcal{A}} \mathbf{P} \cdot d\mathcal{A} = -\int_v \nabla \cdot \mathbf{P} \, dv. \qquad (9\text{-}6)$$

- Since this equation applies any volume chosen as above, the integrands are equal at every point in the dielectric and the **bound volume charge density** is

$$\rho_b = -\nabla \cdot \mathbf{P}. \qquad (9\text{-}7)$$

Polarization Current Density (J_b)

- The motion of bound charges under the action if a time-dependent electric field generates **a polarization current** as

$$I = \frac{dQ_b}{dt} = \frac{|d\mathbf{P}|\, d\mathcal{A}}{dt} \qquad (9\text{-}8)$$

- Thus is, at the given point in space, **P** is a function of time, the motion of bound charge results in a **polarization current density**

$$\mathbf{J}_b = \frac{\partial \mathbf{P}}{\partial t}. \qquad (9\text{-}9)$$

Electric Field of a Polazied Dielectric

- Polarization causes charges to accumulate either at the surface of the dielectric or inside.
- Coulumbs law applies any net charge density regardless of any metter.
- The potential *V* ascribable to the polarized dielectric the same as if the bound charges were located in the vacuum:

$$V = \frac{1}{4\pi\epsilon_0}\int_{v'} \frac{\rho_b \, dv'}{r} + \frac{1}{4\pi\epsilon_0}\int_{\mathscr{A}'} \frac{\sigma_b \, d\mathscr{A}'}{r}, \qquad (9\text{-}10)$$

- If there are also free charges present, then one adds similar integrals for free charges
- The Electric Field is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int_{v'} \frac{\rho\hat{\mathbf{r}}}{r^2} dv' + \frac{1}{4\pi\epsilon_0}\int_{\mathscr{A}'} \frac{\sigma\hat{\mathbf{r}}}{r^2} d\mathscr{A}', \qquad (9\text{-}11)$$

- Where ρ and σ are total charge ensities (free plus bound)

Gauss's Law

- Say a given *v* contains various dielectric, some of which ma be partly inside partly outside. The total free and bound charge within *v* is $Q=Q_f+Q_b$. There is no surface charge on the surface od *v*. Then Gauss's law:

$$\int_{\mathcal{A}} \mathbf{E} \cdot d\boldsymbol{\mathcal{A}} = \frac{Q}{\epsilon_0}. \qquad (9\text{-}13)$$

- If the volume lies enterely inside a dielectric, there is no surface charges and

$$\int_{\mathcal{A}} \mathbf{E} \cdot d\boldsymbol{\mathcal{A}} = \frac{1}{\epsilon_0}\int_v (\rho_f + \rho_b)\, dv = \frac{1}{\epsilon_0}\int_v \rho\, dv, \qquad (9\text{-}14)$$

- Applying divergence theorem, then it yields

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}, \qquad (9\text{-}15)$$

- This is one of the Maxwells four fundamental equations of electromagnetizm

Poissons and Laplace's Equations in Dielectrics

- Since $\boldsymbol{E} = \nabla \boldsymbol{V}$, it follows that

$$\nabla^2 V = -\rho/\epsilon_0. \tag{9-16}$$

- Where $\rho=\rho_f+\rho_b$ is total charge density.
- If the total electric charge density is zero then Laplace's equation applies

$$\nabla^2 V = 0, \tag{9-17}$$

Electric Flux Density (D)

- According to $\nabla . \boldsymbol{E} = \frac{\rho}{\epsilon_o}$,

$$\nabla \cdot \boldsymbol{E} = \frac{\rho_f + \rho_b}{\epsilon_0}. \qquad (9\text{-}18)$$

- We found that ρb=$-\nabla . \boldsymbol{P}$, therefore

$$\nabla \cdot (\epsilon_0 \boldsymbol{E} + \boldsymbol{P}) = \rho_f. \qquad (9\text{-}19)$$

- We conclude a vector

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P} \qquad (9\text{-}20)$$

- This called electric flux density. Thus

$$\nabla \cdot \boldsymbol{D} = \rho_f. \qquad (9\text{-}21)$$

Electric Susceptibility (X_e)

- In most dielectrics **P** is propotional to **E** and points in the same direction. Such dielectrics are linear and isotropic. So in linear and isotropic dielectrics,

$$\boldsymbol{P} = \epsilon_0 \chi_e \boldsymbol{E}, \tag{9-25}$$

- Where X_e is the electric susceptibility of the medium.
- If the dielectric is homogeneous, its susceptibility is independent of the coordinates

Relatve Permittivity (ϵ_r)

- In linear and isotropic dielectric

$$D = \epsilon_0 E + P = \epsilon_0(1 + \chi_e)E = \epsilon_0 \epsilon_r E = \epsilon E, \qquad (9\text{-}26)$$

- Where

$$\epsilon_r = 1 + \chi_e = \frac{\epsilon}{\epsilon_0} \qquad (9\text{-}27)$$

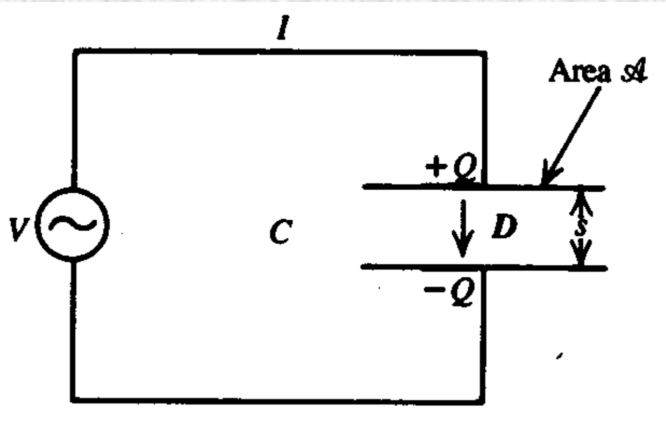
- is the relative permittivity. This quatity is dimentionless and larger than unity. The quantity ϵ is the permittivity and its dimention is the same as ϵ_0. ,Thus, for linear and isotropic dielectrics;

$$P = \epsilon_0 \chi_e E = \epsilon_0(\epsilon_r - 1)E. \qquad (9\text{-}28)$$

Relative Permittivity of Various Materials

TYPE	FREQUENCY 100	10^6	10^{10}
Barium titanate	1250	1140	100
Benzene	2.28	2.28	2.28
Birch (yellow)		2.7	1.95
Butyl rubber	2.43	2.40	2.38
Carbon tetrachloride	2.17	2.17	2.17
Fused silica	3.78	3.78	3.78
Glass (soda borosilicate)	5.0	4.84	4.82
Ice		4.15	3.20
Lucite	3.20	2.63	2.57
Neoprene	6.70	6.26	4.0
Polyethylene	2.26	2.26	2.26
Polystyrene	2.56	2.56	2.54
Sodium chloride		5.90	5.90
Soil (dry loam)		2.59	2.55
Steatite	6.55	6.53	6.51
Styrofoam	1.03	1.03	1.03
Teflon	2.1	2.1	2.08
Water	81	78.2	34
Wheat (red, winter)		4.3	2.6

Displacement Current Density



- Figure shows a paralel-plate capasitor connected to a source of alternatşng voltage. Then

$$I = \frac{V}{Z} = \frac{Es}{1/j\omega C} = j\omega Es \frac{\epsilon_r \epsilon_0 \mathscr{A}}{s} \qquad (9\text{-}46)$$

$$= \mathscr{A} j\omega \epsilon_r \epsilon_0 E = \mathscr{A} j\omega D = \mathscr{A}\frac{dD}{dt} = \frac{dQ}{dt}. \qquad (9\text{-}47)$$

- The displacement current density consists of two parts:

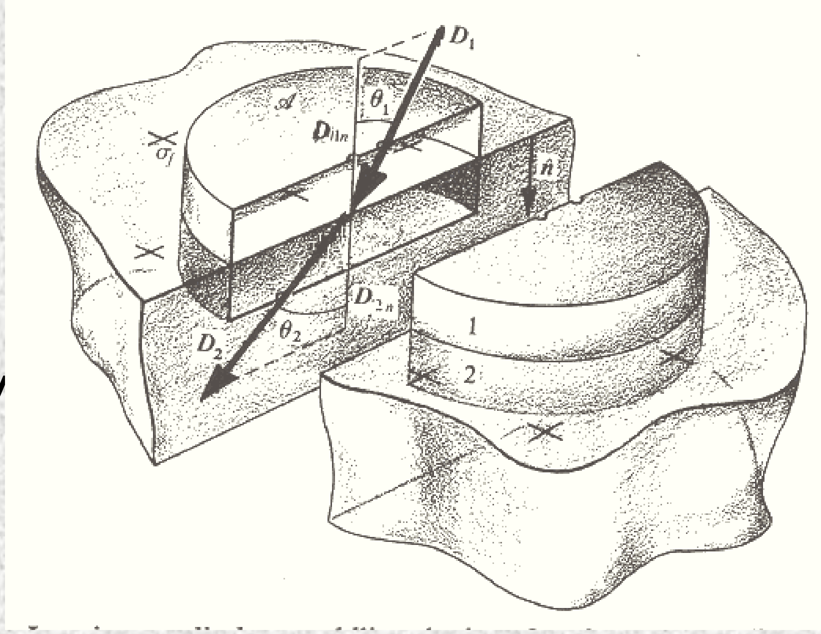
$$\frac{\partial \mathbf{D}}{\partial t} = \frac{\partial}{\partial t}(\epsilon_0 \mathbf{E} + \mathbf{P}) = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \frac{\partial \mathbf{P}}{\partial t}. \qquad (9\text{-}48)$$

- First term is nameless and can exits even in a vacuum. The second term is the polarization current density.

Continuity Conditions at an Interface -1

- **The Potential *V*:** is continuous across the boundary between two media. Othervise a discontinuity would imply an ifinitely large *E* which is physically impossible
- **The Normal Component of D:**

Consider a short imaginary cylindir spanning the interface. Top and bottom faces are paralel and close to the boundary. The interface carries a free Surface charge sensity σ_b. According to Gauss's law



$$(D_{2n} - D_{1n})\mathscr{A} = \sigma_f \mathscr{A}, \qquad (\mathbf{D}_2 - \mathbf{D}_1)\cdot\hat{\mathbf{n}} = \sigma_f, \qquad (10\text{-}9)$$

So normal component of **D** is continuous across the interface

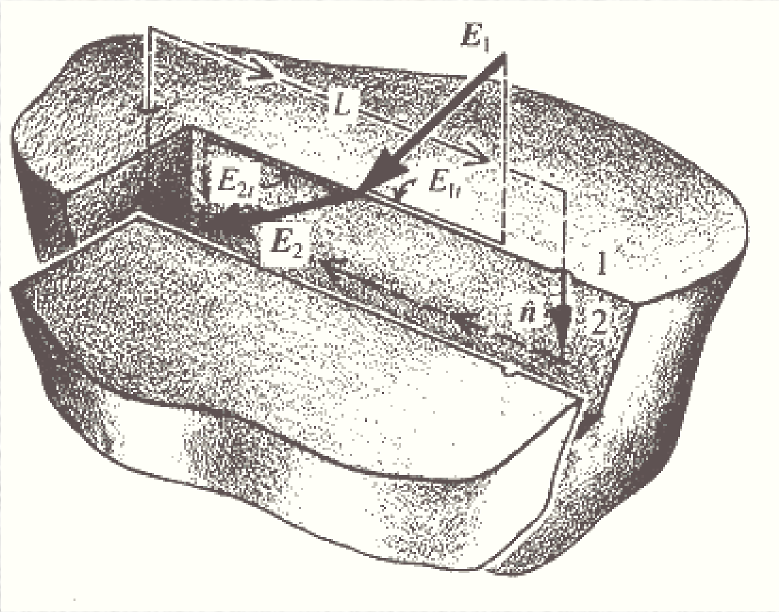
Continuity Conditions at an Interface -2

- **The Tangential Component of E:**

Consider a path with two sides of length L paralel to the boundary and close to it.

If L is too short then **E** does not varry over that distance, Then

$$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = E_{1t}L - E_{2t}L. \qquad (10\text{-}10)$$



This line integral is zero, and thus

$$E_{1t} = E_{2t}†, \quad \text{veya} \quad (\boldsymbol{E}_1 - \boldsymbol{E}_2) \times \hat{\boldsymbol{n}} = 0, \qquad (10\text{-}11)$$

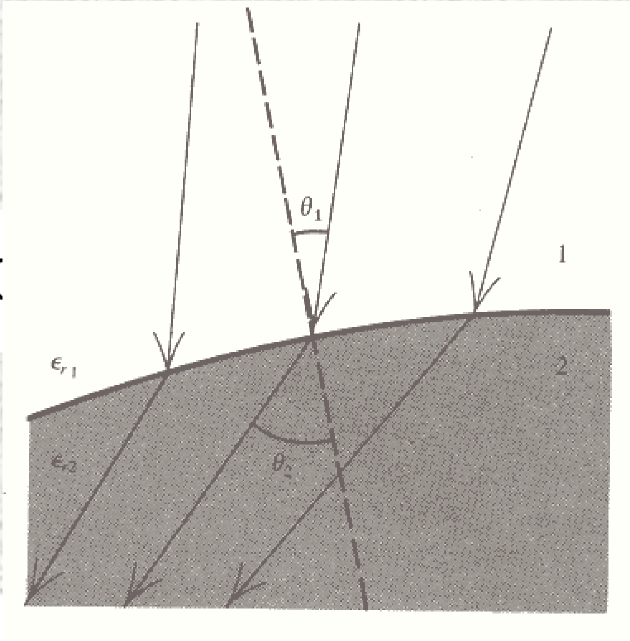
So tangentil component of **E** is continuous across the interface

Bending of Lines of E at the interface

- In figure, the surface charge density is zero
- The normal component of **D** requires

$$D_1 \cos \theta_1 = D_2 \cos \theta_2 \qquad (10\text{-}12)$$

$$\epsilon_{r1}\epsilon_0 E_1 \cos \theta_1 = \epsilon_{r2}\epsilon_0 E_2 \cos \theta_2. \qquad (1$$



- Similary, the tangential component of **E** must satisfy

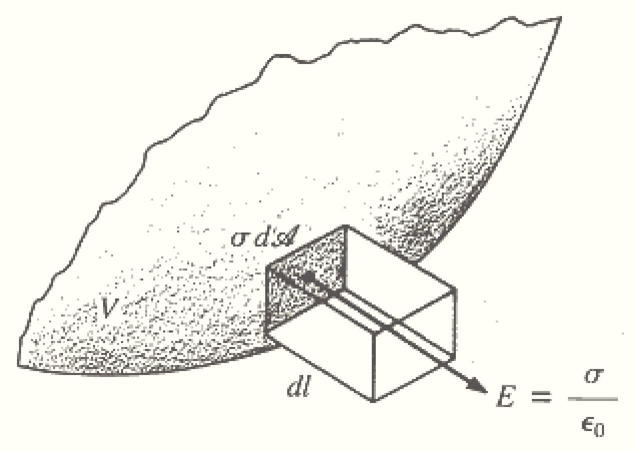
$$E_1 \sin \theta_1 = E_2 \sin \theta_2. \qquad (10\text{-}14)$$

- Dividing the third equation by the second gives

$$\frac{\tan \theta_1}{\epsilon_{r1}} = \frac{\tan \theta_2}{\epsilon_{r2}}. \qquad (10\text{-}15)$$

The Energy Desity Expressed in Terms of E and D

- Say the element of area ***dA*** moves by a distance ***dl*** as in figüre, then the work



$$d\mathcal{E} = (\sigma\, d\mathcal{A})\left(\frac{E}{2}\right) dl, \qquad (10\text{-}25)$$

- The field acting on *σdA* is *E/2*. So

$$d\mathcal{E} = \mathbf{D} \cdot d\mathcal{A} \frac{E}{2} dl = \frac{\mathbf{D} \cdot \mathbf{E}}{2} d\mathcal{A}\, dl = \frac{\mathbf{D} \cdot \mathbf{E}}{2} dv. \qquad (10\text{-}26)$$

- The total work done by the field is

$$\mathcal{E} = \int_v \tfrac{1}{2} \mathbf{E} \cdot \mathbf{D}\, dv, \qquad (10\text{-}27)$$

- The energy density is $\mathcal{E}' = \tfrac{1}{2} \mathbf{E} \cdot \mathbf{D}. \qquad (10\text{-}28)$

- If the dielectric is linear and isotropic $\mathcal{E} = \int_v \frac{\epsilon_r \epsilon_0 E^2}{2} dv.$

Enegy Density Associated with Polarization

- Since

$$\mathscr{E}' = \tfrac{1}{2}\mathbf{E}\cdot\mathbf{D} = \tfrac{1}{2}\mathbf{E}\cdot(\epsilon_0\mathbf{E}+\mathbf{P}) = \tfrac{1}{2}\epsilon_0 E^2 + \tfrac{1}{2}\mathbf{E}\cdot\mathbf{P}, \qquad (10\text{-}30)$$

- The Energy density associated with polarization alone is

$$\frac{1}{2}\mathbf{E}.\,\mathbf{P}$$

- In isotropic dielectrics

$$\mathscr{E}' = \tfrac{1}{2}\epsilon_r\epsilon_0 E^2 = \tfrac{1}{2}(1+\chi_e)\epsilon_0 E^2. \qquad (10\text{-}31)$$