EEE321 Electromagnetic Fileds and Waves

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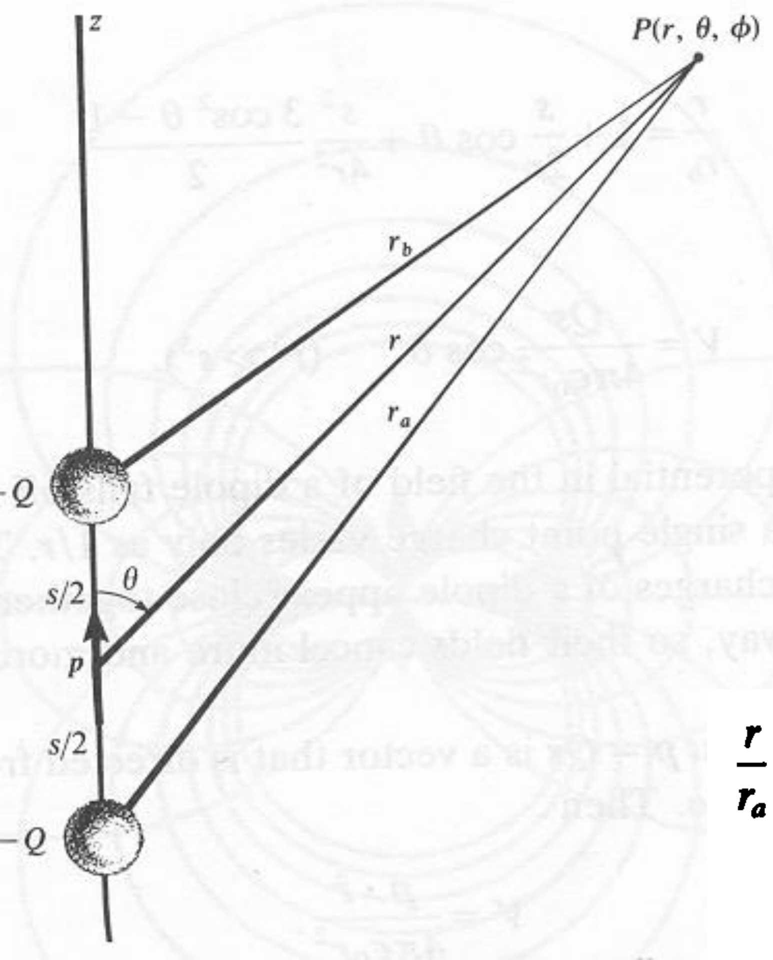
(3rd Week)

Outline

- The Electric Multipoles
- The potential Energy
- The Capacitance
- Electic forces
- Virtual Work

The Electric Dipole-1

- The Electric dipole consists of two charges, one positive and one negative, of the same magnetute, and separated by a distance *s.*



- Let us find *V* and **E** at the point *P* situated at a distance *r* (*r>>s)*

$$V = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right), \tag{5-1}$$

$$r_a^2 = r^2 + \left(\frac{s}{2}\right)^2 + rs\cos\theta. \tag{5-2}$$

- If we divide both sides by r^2 and take the inverse

$$\frac{r}{r_a} = \left[1 + \left(\frac{s}{2r}\right)^2 + \frac{s}{r}\cos\theta\right]^{-1/2} \tag{5-3}$$

$$= 1 - \frac{1}{2}\left(\frac{s^2}{4r^2} + \frac{s}{r}\cos\theta\right) + \frac{3}{8}\left(\frac{s^2}{4r^2} + \frac{s}{r}\cos\theta\right)^2 - \cdots. \tag{5-4}$$

The Electric Dipole-2

- If we neglect the terms of order $(s/r)^3$ and higher, then

$$\frac{r}{r_a} = 1 - \frac{s}{2r}\cos\theta + \frac{s^2}{4r^2}\frac{3\cos^2\theta - 1}{2}. \qquad (5\text{-}5)$$

- Similarly

$$\frac{r}{r_b} = 1 + \frac{s}{2r}\cos\theta + \frac{s^2}{4r^2}\frac{3\cos^2\theta - 1}{2} \qquad (5\text{-}6)$$

$$V = \frac{Qs}{4\pi\epsilon_0 r^2}\cos\theta \qquad (r^3 \gg s^3). \qquad (5\text{-}7)$$

- The dipole moment **p=**Q**s** is a vector that is directed drom the negative to positive charge then

$$V = \frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{4\pi\epsilon_0 r^2}. \qquad (5\text{-}8)$$

The Electric Dipole-3

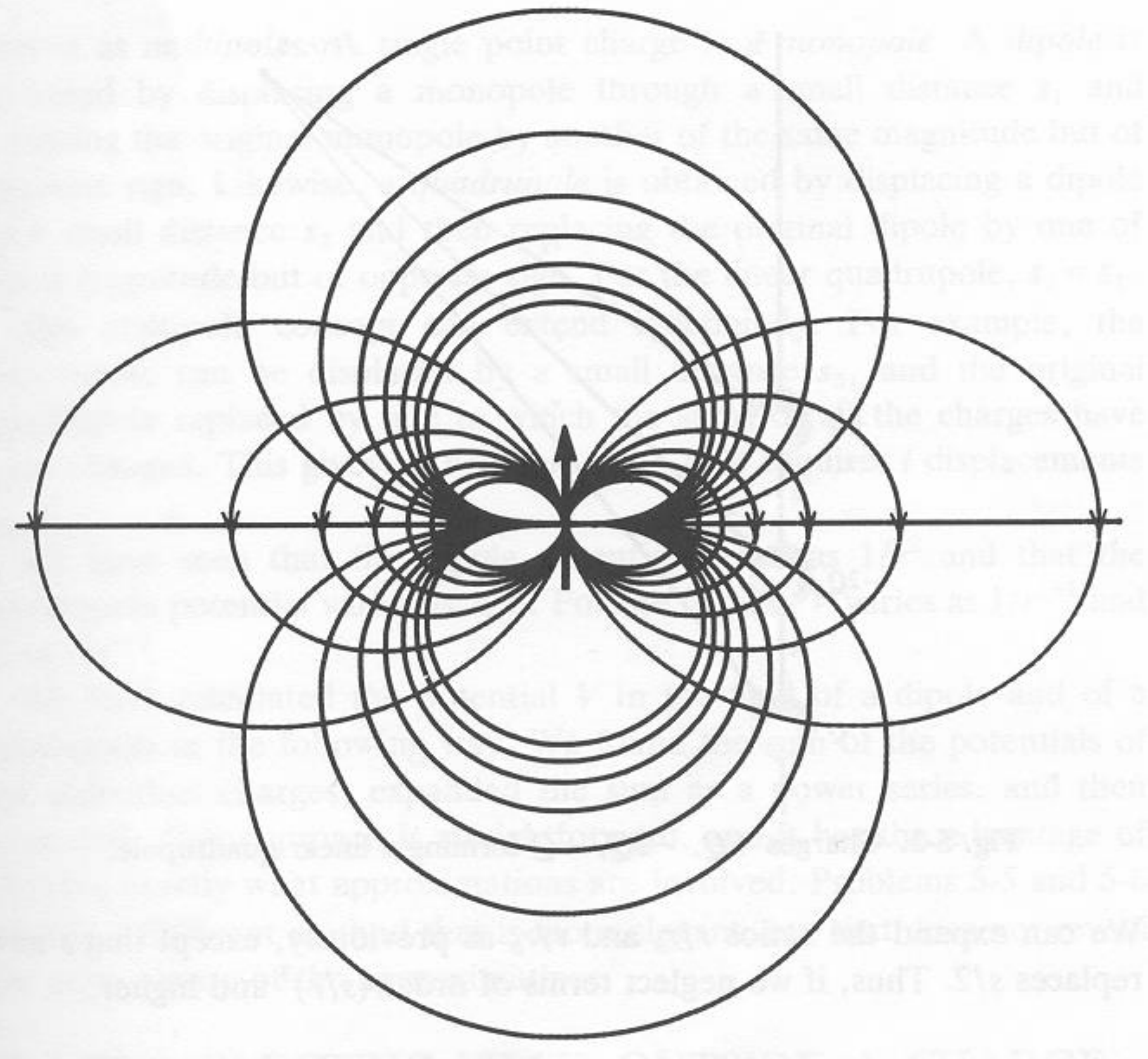
- Let us find the electric field strength **E** in spherical coordinates;

$$E_r = -\frac{\partial V}{\partial r} = \frac{2p\cos\theta}{4\pi\epsilon_0 r^3},$$

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{p\sin\theta}{4\pi\epsilon_0 r^3},$$

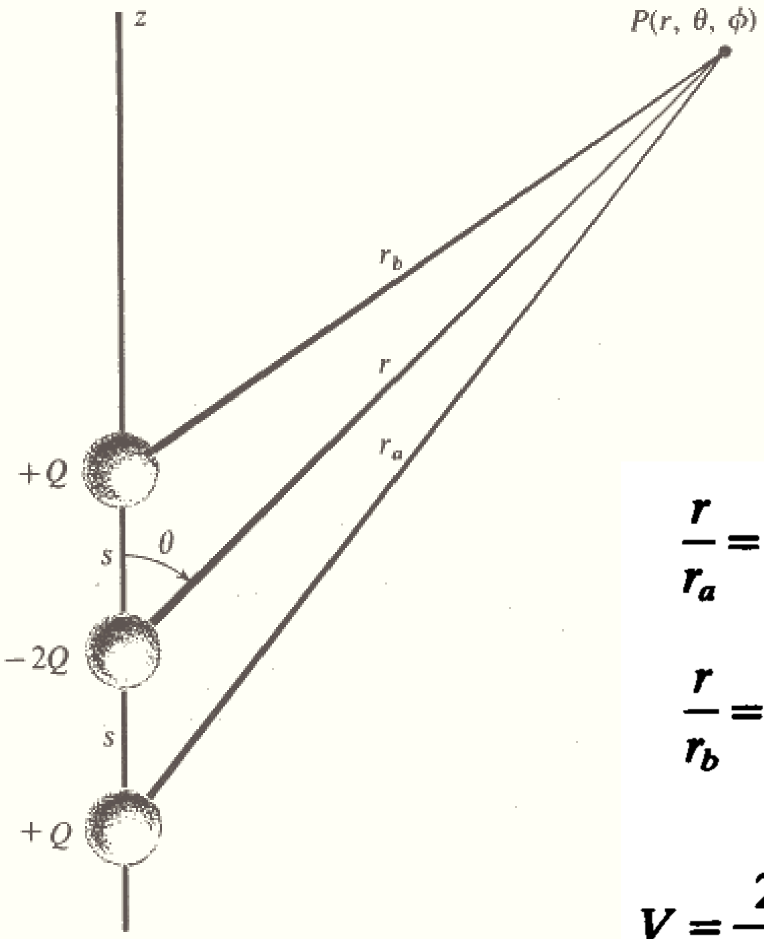
$$E_\phi = -\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi} = 0,$$

$$\boldsymbol{E} = \frac{p}{4\pi\epsilon_0 r^3}(2\cos\theta\,\hat{\boldsymbol{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}).$$



The Linear Electric Quadrupole

- The Linear electric quadrupole set of three charges that are separated by a distance *s and* of the same magnetute.



- Let us find *V* and at the point *P* situated at a distance *r* (*r>>s)*

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r_a} - \frac{2Q}{r} + \frac{Q}{r_b}\right) = \frac{Q}{4\pi\epsilon_0 r}\left(\frac{r}{r_a} + \frac{r}{r_b} - 2\right). \qquad (5\text{-}14)$$

- If we expand r/r_a and r/r_b as the dipole;

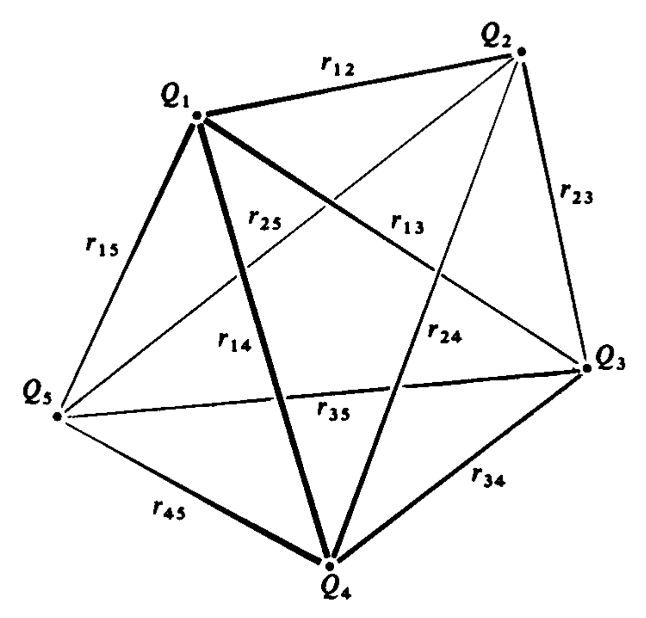
$$\frac{r}{r_a} = 1 - \frac{s}{r}\cos\theta + \frac{s^2}{r^2}\frac{(3\cos^2\theta - 1)}{2}, \qquad (5\text{-}15)$$

$$\frac{r}{r_b} = 1 + \frac{s}{r}\cos\theta + \frac{s^2}{r^2}\frac{(3\cos^2\theta - 1)}{2}, \qquad (5\text{-}16)$$

$$V = \frac{2Qs^2}{4\pi\epsilon_0 r^3}\frac{(3\cos^2\theta - 1)}{2} \qquad (r^3 \gg s^3). \qquad (5\text{-}17)$$

The Potential Energy of a Set of Point Charges-1

- Imagine a set of N point charges distributed in the space



- The total energy of the system is equal to the work performed by the electric forces in the processof dispersing the charges out to infinity
- First, let Q_1 recede to infinity slowly, keeping the electric and mechanical forces in equilibrium.
- The decrease in potential energy is equal to Q_1 multiplied by the potential V_1

$$\mathscr{E}_1 = \frac{Q_1}{4\pi\epsilon_0}\left(\frac{Q_2}{r_{12}} + \frac{Q_3}{r_{13}} + \cdots + \frac{Q_N}{r_{1N}}\right). \qquad (6\text{-}1)$$

- Similarly

$$\mathscr{E}_2 = \frac{Q_2}{4\pi\epsilon_0}\left(\frac{Q_3}{r_{23}} + \frac{Q_4}{r_{24}} + \cdots + \frac{Q_N}{r_{2N}}\right). \qquad (6\text{-}2)$$

The Potential Energy of a Set of Point Charges-2

- The total potential energy

$$\mathscr{E} = \mathscr{E}_1 + \mathscr{E}_2 + \mathscr{E}_3 + \cdots \mathscr{E}_N$$

$$= \frac{Q_1}{4\pi\epsilon_0}\left(0 + \frac{Q_2}{r_{12}} + \frac{Q_3}{r_{13}} + \frac{Q_4}{r_{14}} + \cdots + \frac{Q_N}{r_{1N}}\right)$$

$$+ \frac{Q_2}{4\pi\epsilon_0}\left(0 + 0 + \frac{Q_3}{r_{23}} + \frac{Q_4}{r_{24}} + \cdots + \frac{Q_N}{r_{2N}}\right)$$

$$+ \frac{Q_3}{4\pi\epsilon_0}\left(0 + 0 + 0 + \frac{Q_4}{r_{34}} + \cdots + \frac{Q_N}{r_{3N}}\right) + \cdots$$

$$+ \frac{Q_N}{4\pi\epsilon_0}(0 + 0 + 0 + 0 + \cdots + 0).$$

(6-

$$2\mathscr{E} = \frac{Q_1}{4\pi\epsilon_0}\left(0 + \frac{Q_2}{r_{12}} + \frac{Q_3}{r_{13}} + \frac{Q_4}{r_{14}} + \cdots \frac{Q_N}{r_{1N}}\right)$$

$$+ \frac{Q_2}{4\pi\epsilon_0}\left(\frac{Q_1}{r_{21}} + 0 + \frac{Q_3}{r_{23}} + \frac{Q_4}{r_{24}} + \cdots + \frac{Q_N}{r_{2N}}\right)$$

$$+ \frac{Q_3}{4\pi\epsilon_0}\left(\frac{Q_1}{r_{31}} + \frac{Q_2}{r_{32}} + 0 + \frac{Q_4}{r_{34}} + \cdots + \frac{Q_N}{r_{3N}}\right) + \cdots$$

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$$+ \frac{Q_N}{4\pi\epsilon_0}\left(\frac{Q_1}{r_{N1}} + \frac{Q_2}{r_{N2}} + \frac{Q_3}{r_{N3}} + \frac{Q_4}{r_{N4}} + \cdots + 0\right). \qquad (6\text{-}5)$$

$$2\mathscr{E} = Q_1V_1 + Q_2V_2 + Q_3V_3 + \cdots + Q_NV_N, \qquad (6\text{-}6)$$

$$\mathscr{E} = \frac{1}{2}\sum_{i=1}^{N} Q_iV_i. \qquad (6\text{-}7)$$

- This energy which does not include the energy required to assemble the individual charhes themselves, can be positive, negative or zero.

The Potential Energy of a Continuous Charge Distribution

- For the continuous electric charge distribution, we replace Qi by ρdv and thr summation by an integration over any volüme that contains all the charge

$$\mathscr{E} = \frac{1}{2}\int_{v} V\rho \, dv. \qquad (6\text{-}8)$$

- This integral is equal to the work performed by the electric forces in going from the given charge distributioun to the situation where $\rho = 0$ everywhere
- If there are surface charge density σ, then their stored energy is

$$\mathscr{E} = \frac{1}{2}\int_{\mathscr{A}} \sigma V \, d\mathscr{A}, \qquad (6\text{-}9)$$

- Where *A* includes all surface carrying charge

The Potential energy expressed in Terms of E

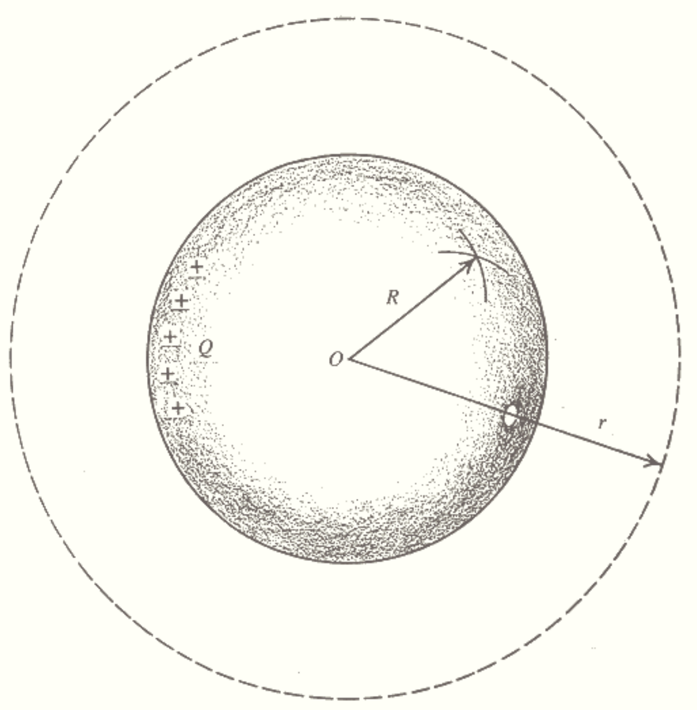
- Uptil now the potential energy is expressed in terms of the charge density and the potential. Both the charge density and the potential are related to **E**. Then we shall find that

$$\mathscr{E} = \int_v \frac{\epsilon_0 E^2}{2} dv, \qquad (6\text{-}11)$$

- Where the volüme *v* includes all the regions where **E** exits.

Example-1

- Let ua find the potential energy of the conducting sphere of Radius R carring a charge Q



- **First Method**: The whole charge Q is at the potential $\frac{Q}{4\Pi\epsilon_o R}$. Then

$$\mathscr{E} = \frac{1}{2} Q \frac{Q}{4\pi\epsilon_0 R} = \frac{Q^2}{8\pi\epsilon_0 R}. \qquad (6\text{-}12)$$

- **Second Method**: Imagine that the Radius of the charged spherical conductor increases slowly and eventually becomes infinity

$$d\mathscr{E} = (\epsilon_0 E \, d\mathscr{A})\left(\frac{E}{2}\right) dR = \frac{\epsilon_0 E^2}{2} dv, \qquad (6\text{-}13)$$

$$\mathscr{E} = \int_R^\infty \frac{\epsilon_0 E^2}{2} dv = \int_R^\infty \frac{\epsilon_0}{2} \left(\frac{Q}{4\pi\epsilon_0 r^2}\right)^2 4\pi r^2 \, dr = \frac{Q^2}{8\pi\epsilon_0 R} \qquad (6\text{-}14)$$

- **Third Method**:Let *A(r)* be the area of any surface of Radius *r*, concentric with the conducting sphere and outside it. Then from Gauss's law

$$\mathscr{E} = \tfrac{1}{2}QV = \tfrac{1}{2}\int_{\mathscr{A}(r)} \epsilon_0 E \, d\mathscr{A} \int_R^{\infty} E \, dr. \qquad (6\text{-}15)$$

$$\mathscr{E} = \tfrac{1}{2}\int_R^{\infty}\left(\int_{\mathscr{A}(r)} \epsilon_0 E \, d\mathscr{A}\right) E \, dr \qquad (6\text{-}16)$$

$$= \tfrac{1}{2}\int_R^{\infty} (4\pi r^2 \epsilon_0 E) E \, dr = \int_R^{\infty} \frac{\epsilon_0 E^2}{2} 4\pi r^2 \, dr \qquad (6\text{-}17)$$

$$= \int_R^{\infty} \frac{\epsilon_0 E^2}{2} dv = \frac{Q^2}{8\pi\epsilon_0 R}, \qquad (6\text{-}18)$$

The Capacitance of an Isolated Conductor

- Imagine a finite conductor situated a long distance from any other bady and carrying a charge *Q.* If *Q* changes, the conductor's potentials also changes. The ratio called **capacitance** constant.

$$C = \frac{Q}{V}. \qquad (6\text{-}19)$$

- The energy stored in the field of an isolated coductor is;

$$\mathscr{E} = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}. \qquad (6\text{-}20)$$

- The capacitance depends solely on the size and shape of the conductor.

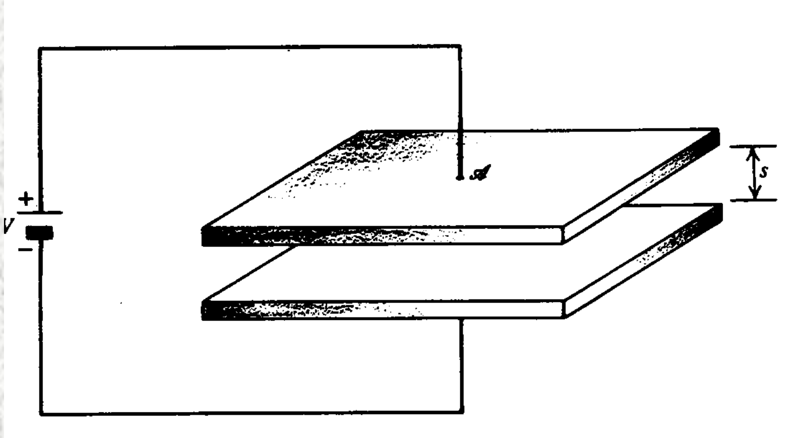
Example-2

- If an isolated conducting sphere of redius R carries a charge, the potentialat its surface is $\frac{Q}{4\Pi\epsilon_o R}$ and

$$C = 4\pi\epsilon_0 R = 1.11 \times 10^{-10} R \quad \text{farad}$$
$$= 111R \quad \text{picofarads.} \qquad (6\text{-}21)$$

Example-3

- A paralel plate capasitor considt of two conducting plates of area *A*, separated by a distance *s*. The plates carry charges *Q* and –*Q*. We neglect edge effects.



- From Gauss's law

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\mathscr{A}\epsilon_0}, \qquad V = \frac{Qs}{\mathscr{A}\epsilon_0}. \qquad (6\text{-}22)$$

$$C = \frac{\epsilon_0 \mathscr{A}}{s}. \qquad (6\text{-}23)$$

- The stored energy

$$\mathscr{E} = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C} = \frac{Q^2 s}{2\epsilon_0 \mathscr{A}}, \qquad (6\text{-}24)$$

- or

$$\mathscr{E} = \int \frac{\epsilon_0 E^2}{2} dv = \frac{\epsilon_0}{2}\left(\frac{Q}{\epsilon_0 \mathscr{A}}\right)^2 \mathscr{A}s = \frac{Q^2 s}{2\epsilon_0 \mathscr{A}}. \qquad (6\text{-}25)$$

Example-4

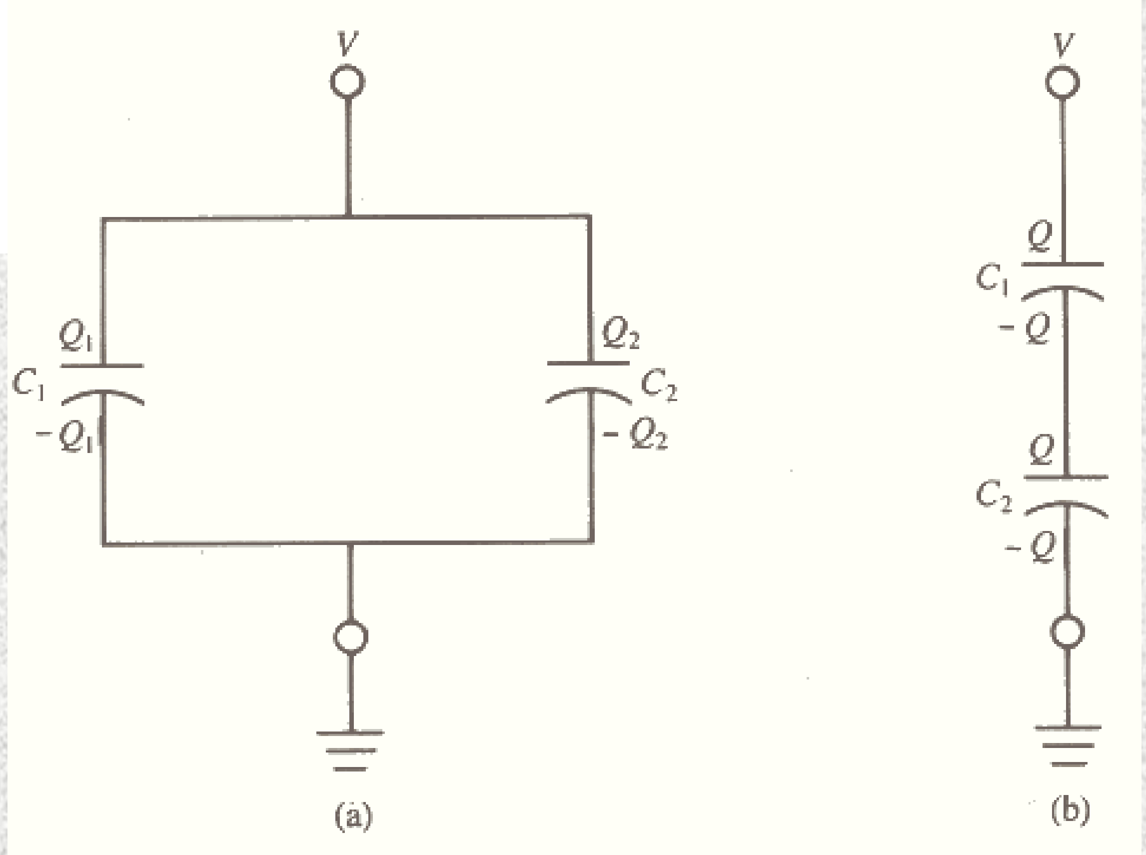
- (a) Capacitors connected in paralel share the same voltage

$$C = \frac{Q_1 + Q_2}{V} = \frac{Q_1}{V} + \frac{Q_2}{V} = C_1 + C_2. \qquad (6\text{-}26)$$

- (b) Capacitors connected in series carry the same charges

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} = \frac{Q}{C},$$

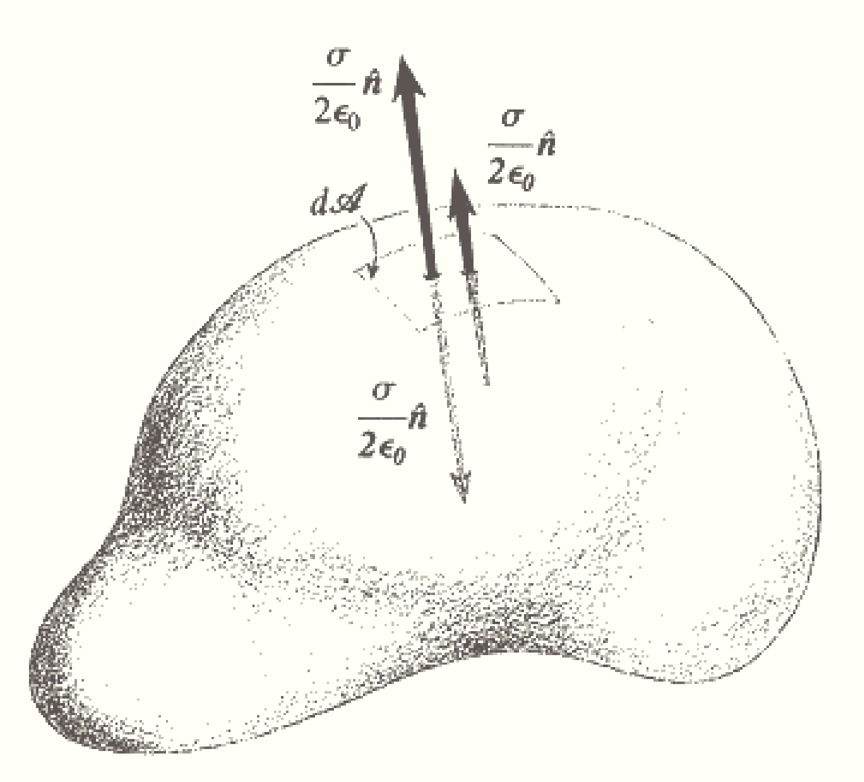
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}, \qquad C = \frac{C_1 C_2}{C_1 + C_2}.$$



Electric Forces on Conductors

- To calculate the magnitute of the electric force, consider a conductor carrying a surface charge density σ with the electric field strength **E** near the surface.
- The force on the element of area *dA* of the conductor is

$$dF = \frac{\sigma}{2\epsilon_0}\sigma\, d\mathcal{A} = \frac{\sigma^2}{2\epsilon_0}\, d\mathcal{A}.$$



- The surface force density is

$$F' = \frac{dF}{d\mathcal{A}} = \frac{\sigma^2}{2\epsilon_0} = \frac{\epsilon_0 E^2}{2} \qquad \text{newtons/meter}^2.$$

- The force per unit area on the conductor is equal to the the Energy density in the field.

The net force;

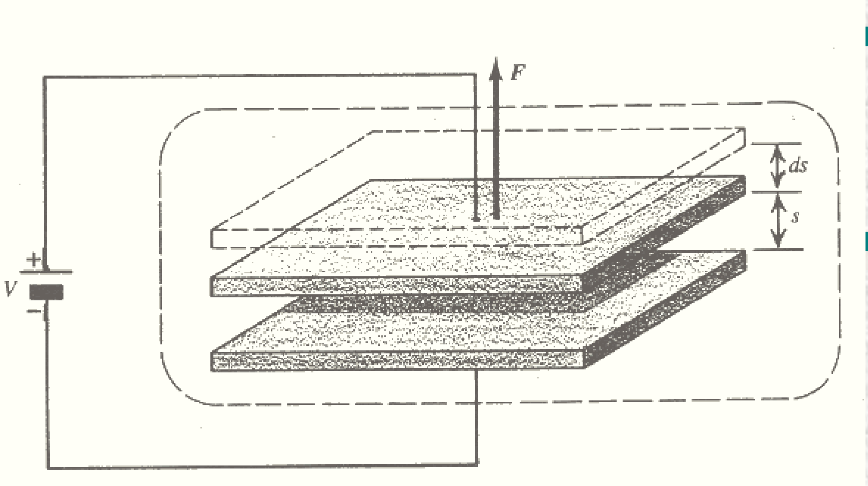
$$F = \frac{\epsilon_0}{2}\oint_{\mathcal{A}} E^2\, d\mathcal{A}, \qquad (6\text{-}31)$$

Calculating Electric Forces by the Method of Virtual Work

- We can also calculate electric force by the method of virtual work.
- Tis method consists in postulating an infinitesimal displacement and then applying the principle of conservation of energy
- First, we deifne the system, and then we calculate the energy fed into it in the course of the displacement
- The energy is equal to the increase in the internal energy pf the system

Example-5

- The parallel-plate capacitor is connected to a battary supplying a fixed voltage



- Assume that the distance *s* increases by *ds*, and apply the principles of the virtual work
- The increase in the electric energy of the capacitor is

$$\mathscr{E}_F + \mathscr{E}_B = \mathscr{E}_E, \qquad (6\text{-}32)$$

$$\mathscr{E}_F = F\,ds, \qquad \mathscr{E}_B = V\,dQ, \qquad (6\text{-}33)$$

$$\mathscr{E}_B = V(V\,dC) = V^2 d\frac{\epsilon_0 \mathscr{A}}{s} = -V^2 \epsilon_0 \mathscr{A}\frac{ds}{s^2} = -(\epsilon_0 E^2)(\mathscr{A}\,ds). \qquad (6\text{-}34)$$

$$\mathscr{E}_E = d\left(\frac{\epsilon_0 E^2}{2}\mathscr{A}s\right) = d\left(\frac{\epsilon_0 V^2 \mathscr{A}s}{2s^2}\right) = \epsilon_0\frac{\mathscr{A}V^2}{2}\left(-\frac{ds}{s^2}\right) = -\frac{\epsilon_0 E^2}{2}\mathscr{A}\,ds. \qquad (6\text{-}35)$$

$$F\,ds - \epsilon_0 E^2 \mathscr{A}\,ds = -\frac{\epsilon_0 E^2}{2}\mathscr{A}\,ds, \qquad (6\text{-}36)$$

$$F = \frac{\epsilon_0 E^2}{2}\mathscr{A}, \qquad (6\text{-}37)$$