EEE321 Electromagnetic Fields and Waves

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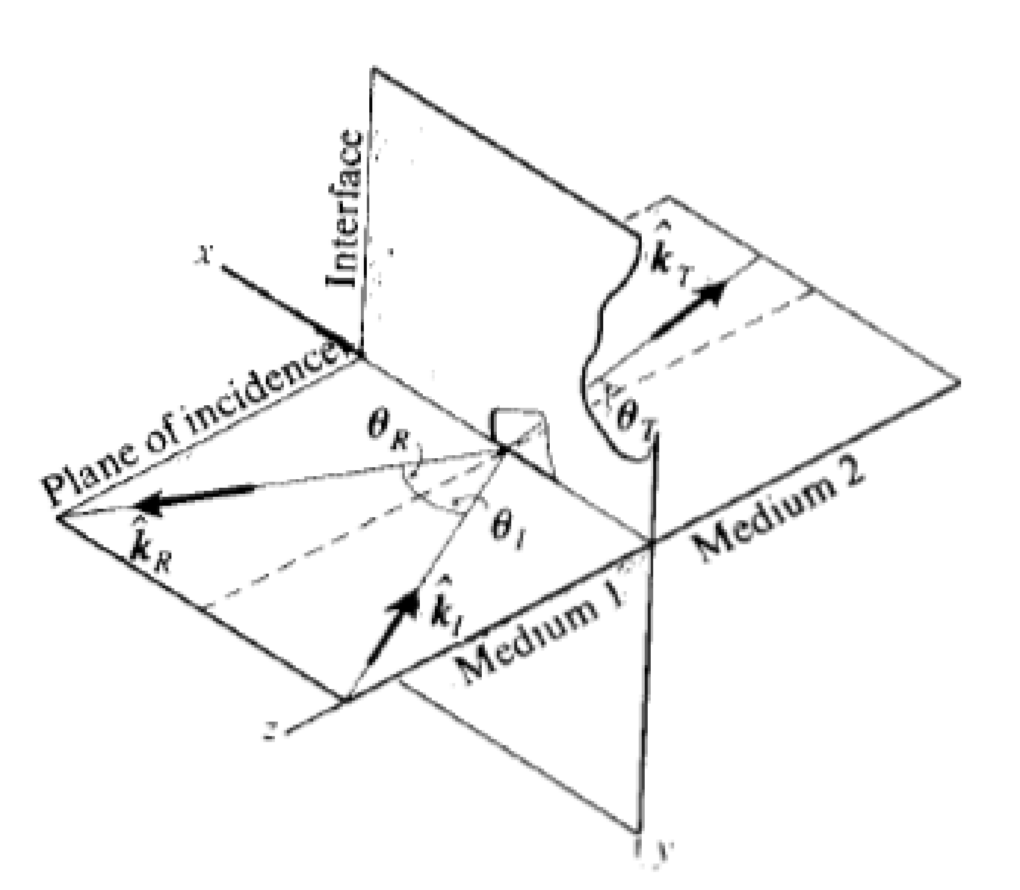
(12th Week)

Outline

- Reflection and Refraction
- Snell's Law
- Fresnel's Equations
- Reflection and refraction at the interface between two nonmagnetic nonconductors
- Brewster Angle
- Coefficients of reflection R and of tranmission T

Assumptions

- (1) the media extends to infinity on either side of the interface. This avoids mutiple reflection.
- (2) The media are homogeneous, isotropic, linear and stationary (HILS) and lossless.
- (3) The interface is infinitely thin
- (4) the incident wave is plane and uniform



Reflection and Refraction

- Medium 1 carries the incident and reflected waves.
- Madium 2 carries the refracted wave.
- For simplicity, the incident wave is linearly polarized. Then incident wave,

$$\boldsymbol{E}_I = \boldsymbol{E}_{Im} \exp j(\omega_I t - \boldsymbol{k}_I \cdot \boldsymbol{r}), \qquad (30\text{-}1)$$

- Where the vector wave numver $\mathbf{k_I}$ is real and points the direction of propogation of incident wave
- For convenince, we set the origin of $\boldsymbol{r}$ in the interface and we take $\boldsymbol{E_{Im}}$ to be real
- Since the incident is plane, all the incident rays are paralel.
- By hypotesis, the interface is plane, we expect the reflected and transmitted waves to be the form

$$\boldsymbol{E}_R = \boldsymbol{E}_{Rm} \exp j(\omega_R t - \boldsymbol{k}_R \cdot \boldsymbol{r}), \qquad (30\text{-}2)$$

$$\boldsymbol{E}_T = \boldsymbol{E}_{Tm} \exp j(\omega_T t - \boldsymbol{k}_T \cdot \boldsymbol{r}). \qquad (30\text{-}3)$$

- From the wave equation applied to medium 1, with σ=0, ρ_f=0,

$$\nabla^2 \mathbf{E}_R + \epsilon_1 \mu_1 \omega^2 \mathbf{E}_R = \nabla^2 \mathbf{E}_R + k_1^2 \mathbf{E}_R = 0, \qquad (30\text{-}4)$$

- Where

$$k_1 = \frac{1}{\bar{\lambda}_1} = \frac{n_1}{\bar{\lambda}_0} = n_1 k_0 = \omega(\epsilon_1 \mu_1)^{1/2}. \qquad (30\text{-}5)$$

- A similar string of equation applies to k_2. Also,

$$k_{Ix}^2 + k_{Iy}^2 + k_{Iz}^2 = k_{Rx}^2 + k_{Ry}^2 + k_{Rz}^2 = k_1^2, \qquad k_{Tx}^2 + k_{Ty}^2 + k_{Tz}^2 = k_2^2. \quad (30\text{-}6)$$

- The wave numbers k_1 and k_2 are real, but $\mathbf{k_R}$, $\mathbf{k_T}$ are vectors that can be complex.
- The tangential componens of **E** is continuous at the interface. This means that the tangential component of $\boldsymbol{E_I}+\boldsymbol{E_R}$ in the mrdium 1, at the interface, is equal to the tangantial component of $\boldsymbol{E_T}$.
- Since three wave are of the same frequency, then

$$\omega_I = \omega_R = \omega_T. \qquad (30\text{-}7)$$

- If k_{Iy} =0 then $k_{Ry} = 0, \qquad k_{Ty} = 0, \qquad (30\text{-}9)$
- And $\mathbf{k_I}$, $\mathbf{k_R}$, $\mathbf{k_T}$ are coplanar. The x components of the **k**’s are thus all equal

$$k_{Rx} = k_{Tx} = k_{Ix} = k_1 \sin\theta_I, \qquad (30\text{-}10)$$

- Where θ_I is the angle of incidence. It is now easy to find $\mathbf{k}_R$:

$$k_{Rx}^2 + k_{Rz}^2 = k_{Ix}^2 + k_{Iz}^2 = k_1^2 \qquad (30\text{-}11)$$

$$k_{Rz}^2 = k_{Iz}^2, \qquad k_{Rz} = -k_{Iz}. \qquad (30\text{-}12)$$

- Then $\mathbf{k_R}$ is also real, the reflected wave is uniform, abd

$$\theta_I = \theta_R. \qquad (30\text{-}13)$$

Snell's Law

- Equation $k_{Rx} = k_{Tx} = k_{Ix} = k_1 \sin\theta_I,$ says that,

$$k_{Tx} = k_1 \sin\theta_I. \qquad (30\text{-}14)$$

- Then

$$k_{Tz}^2 = k_2^2 - k_{Tx}^2 = k_2^2 - k_1^2 \sin^2\theta_I = k_0^2(n_2^2 - n_1^2 \sin^2\theta_I). \qquad (30\text{-}15)$$

- If the term in paranthesis is negative, then there is total reflection. If θ_T is the angle of refraction

$$k_{Tz} = -k_2 \cos\theta_T, \qquad k_{Tz} = k_2 \sin\theta_T. \qquad (30\text{-}16)$$

- From equation 30.14 and 30.16,

$$k_2 \sin\theta_T = k_1 \sin\theta_I, \qquad \text{or} \qquad n_2 \sin\theta_T = n_1 \sin\theta_I. \qquad (30\text{-}17)$$

- This is called Snell's law. The law of reflection and Snell's law are general. Therefore we find that

$$\mathbf{E}_I = \mathbf{E}_{Im} \exp j[\omega t - k_1(x \sin\theta_I - z\cos\theta_I)], \qquad (30\text{-}18)$$

$$\mathbf{E}_R = \mathbf{E}_{Rm} \exp j[\omega t - k_1(x \sin\theta_I + z\cos\theta_I)], \qquad (30\text{-}19)$$

$$\mathbf{E}_T = \mathbf{E}_{Tm} \exp j[\omega t - k_2(x \sin\theta_T - z\cos\theta_T)]. \qquad (30\text{-}20)$$

Fresnel's Equation

- The conditions of continuity at the interface require that

$$E_{Ix} + E_{Rx} = E_{Tx}, \qquad E_{Iy} + E_{Ry} = E_{Ty}, \qquad (30\text{-}21)$$

$$H_{Ix} + H_{Rx} = H_{Tx}, \qquad H_{Iy} + H_{Ry} = H_{Ty}. \qquad (30\text{-}22)$$

- Since the relation

$$\boldsymbol{H} = \frac{\boldsymbol{k} \times \boldsymbol{E}}{\omega\mu} \qquad (30\text{-}23)$$

- applies to all three waves.
- We first find $\mathbf{E_R}$ and $\mathbf{E_T}$ then deduce $\mathbf{H_R}$ and $\mathbf{H_T}$.
- For convenience, it is divided into two parts
  - **E** vectors normal to the plane of incidence
  - **E** vectors paralel to the plane of incidence

E Normal to the Plane of Incidence

- The continuity of the tangential component of **E** at the interface requires that

$$E_{Im} + E_{Rm} = E_{Tm} \quad (30\text{-}24)$$

- At any points on the interface. Similarly the tangential component of **H**,

$$H_{Im} \cos \theta_I - H_{Rm} \cos \theta_I = H_{Tm} \cos \theta_T \quad (30\text{-}25)$$

- Since

$$Z = \frac{E}{H} = \frac{\omega\mu}{k} = \frac{\omega\mu}{nk_0} = \frac{\omega\mu}{n(\omega/c)} = \frac{c\mu}{n}, \quad (30\text{-}27)$$

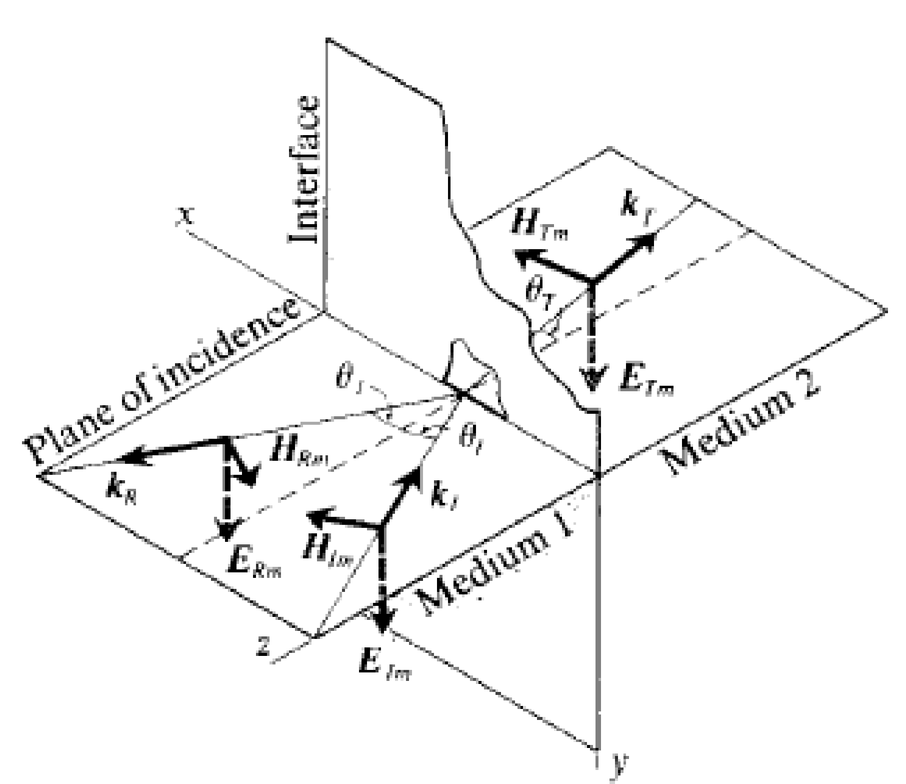
- Then

$$\frac{(E_{Im} - E_{Rm}) \cos \theta_I}{Z_1} = \frac{E_{Tm} \cos \theta_T}{Z_2}, \quad (30\text{-}26)$$

- So, firs two Fresnell's Eq.,

$$\left(\frac{E_{Rm}}{E_{Im}}\right)_\perp = \frac{Z_2 \cos \theta_I - Z_1 \cos \theta_T}{Z_2 \cos \theta_I + Z_1 \cos \theta_T}, \quad (30\text{-}28)$$

$$\left(\frac{E_{Tm}}{E_{Im}}\right)_\perp = \frac{2Z_2 \cos \theta_I}{Z_2 \cos \theta_I + Z_1 \cos \theta_T}. \quad (30\text{-}29)$$



E Normal to the Plane of Incidence

- The **E**'s are now all in the plane of incidence and

$$H_{Im} - H_{Rm} = H_{Tm}, \qquad (30\text{-}30)$$

$$\frac{E_{Im} - E_{Rm}}{Z_1} = \frac{E_{Tm}}{Z_2}. \qquad (30\text{-}31)$$

$$(E_{Im} + E_{Rm}) \cos\theta_I = E_{Tm} \cos\theta_T. \qquad (30\text{-}32)$$

- Then

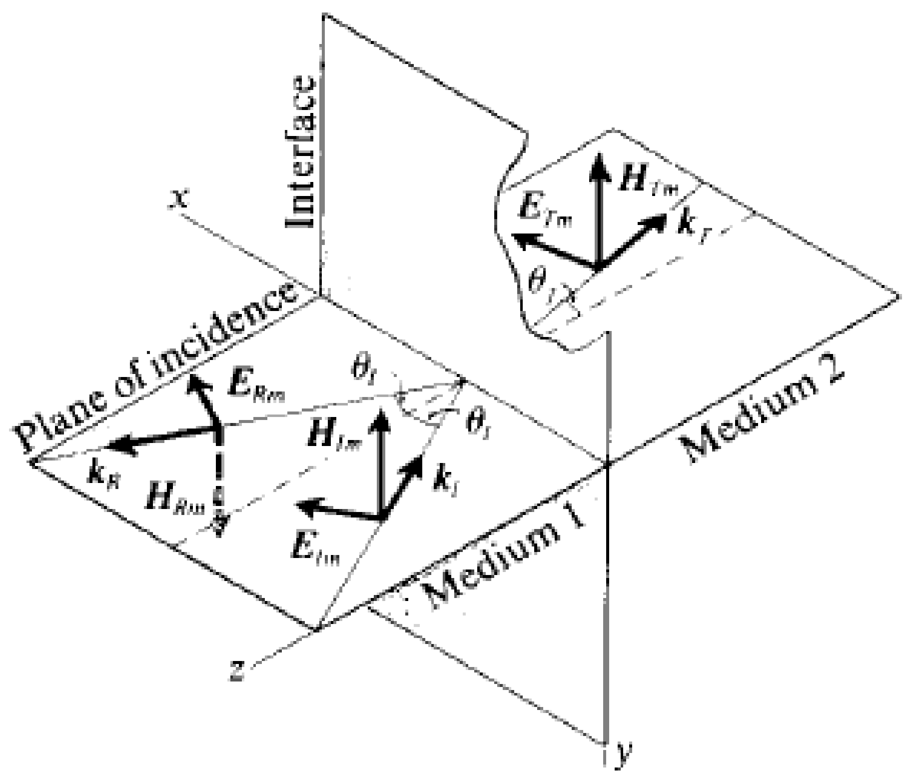
$$\left(\frac{E_{Rm}}{E_{Im}}\right)_{\parallel} = \frac{Z_2 \cos\theta_T - Z_1 \cos\theta_I}{Z_2 \cos\theta_T + Z_1 \cos\theta_I}, \qquad (30\text{-}33)$$

$$\left(\frac{E_{Tm}}{E_{Im}}\right)_{\parallel} = \frac{2Z_2 \cos\theta_I}{Z_2 \cos\theta_T + Z_1 \cos\theta_I}. \qquad (30\text{-}34)$$

- This is the second pair of Fresnell's equations.
- If $\theta_\iota = \theta_R = \theta_T = 0$ then

$$\frac{E_{Rm}}{E_{Im}} = \frac{Z_2 - Z_1}{Z_2 + Z_1}, \qquad (30\text{-}35)$$

$$\frac{E_{Tm}}{E_{Im}} = \frac{2Z_2}{Z_2 + Z_1}. \qquad (30\text{-}36)$$



Reflection and Refraction at the Interface between two Nonmagnetic Nonconductors

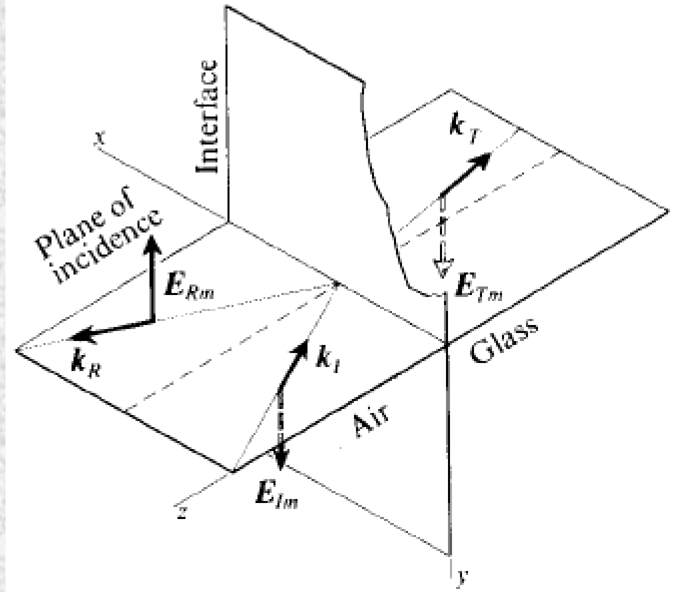
- We still disgard total reflection
- For nonmagnetic nonconductors,

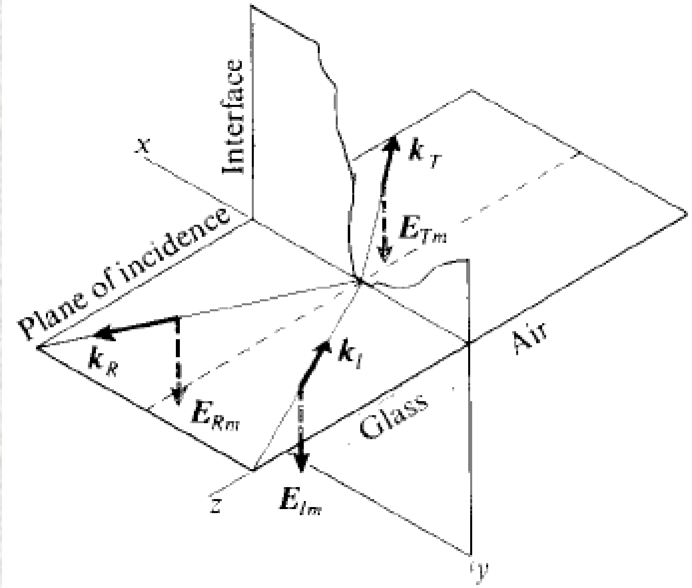
$$Z_1 = \frac{\omega\mu_0}{k_1} = \frac{c\mu_0}{n_1}, \qquad Z_2 = \frac{c\mu_0}{n_2}, \tag{30-37}$$

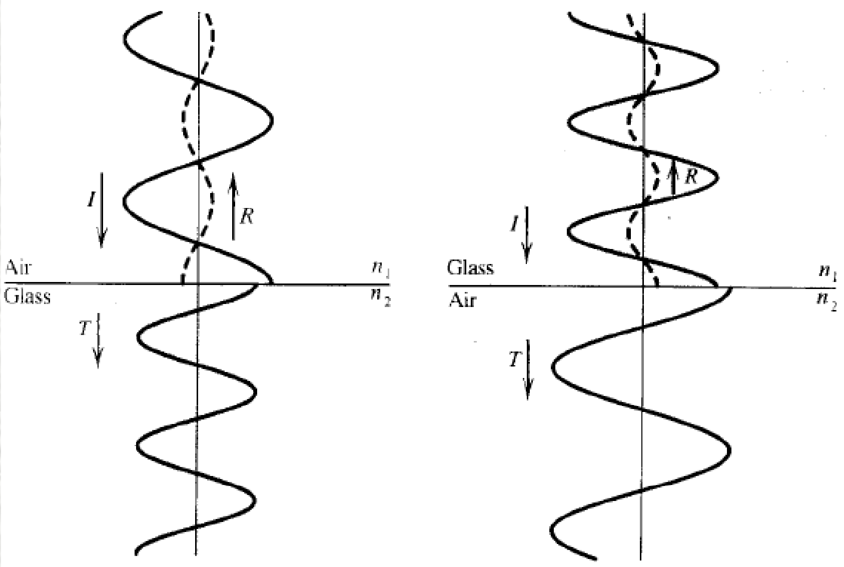
$$\left(\frac{E_{Rm}}{E_{Im}}\right)_\perp = \frac{(n_1/n_2)\cos\theta_I - \cos\theta_T}{(n_1/n_2)\cos\theta_I + \cos\theta_T}, \tag{30-38}$$

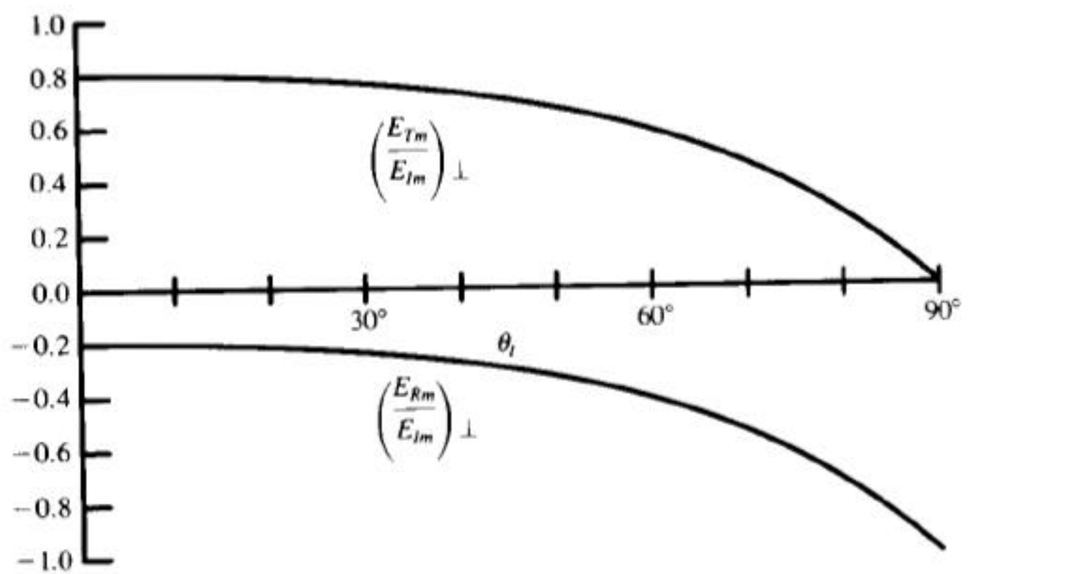
$$\left(\frac{E_{Tm}}{E_{Im}}\right)_\perp = \frac{2(n_1/n_2)\cos\theta_I}{(n_1/n_2)\cos\theta_I + \cos\theta_T}. \tag{30-39}$$

- If $n_1/n_2 < 1$ then $\theta_T < \theta_I$ and $\cos\theta_I < \cos\theta_T$
- If $n_1/n_2 > 1$ then $\theta_T > \theta_I$ and $\cos\theta_I > \cos\theta_T$
- The reflected wave is either π radians out of phase with the incident wave at the interface if $n_1<n_2$, or in phase if $n_1>n_2$









Reflection and refraction when $n_1/n_2 = 1/1.5$, for example, when light falls on a glass of $n = 1.5$. The $\boldsymbol{E}$ field is *normal* to the plane of incidence.

- For an incident wave polarized with its ***E*** vector paralel to the plane of incidence

$$\left(\frac{E_{Rm}}{E_{Im}}\right)_{\parallel} = \frac{-\cos\theta_I + (n_1/n_2)\cos\theta_T}{\cos\theta_I + (n_1/n_2)\cos\theta_T}, \quad (30\text{-}40)$$

$$\left(\frac{E_{Tm}}{E_{Im}}\right)_{\parallel} = \frac{2(n_1/n_2)\cos\theta_I}{\cos\theta_I + (n_1/n_2)\cos\theta_T}. \quad (30\text{-}41)$$

- Second ratio is always positive. Then E_{Tm} and E_{Im} are in phase at the interface
- E_{Tm} and E_{Im} can be either in phase or π radius out of phase.
- They are in phase if

$$\frac{n_1}{n_2}\cos\theta_T > \cos\theta_I, \quad (30\text{-}42)$$

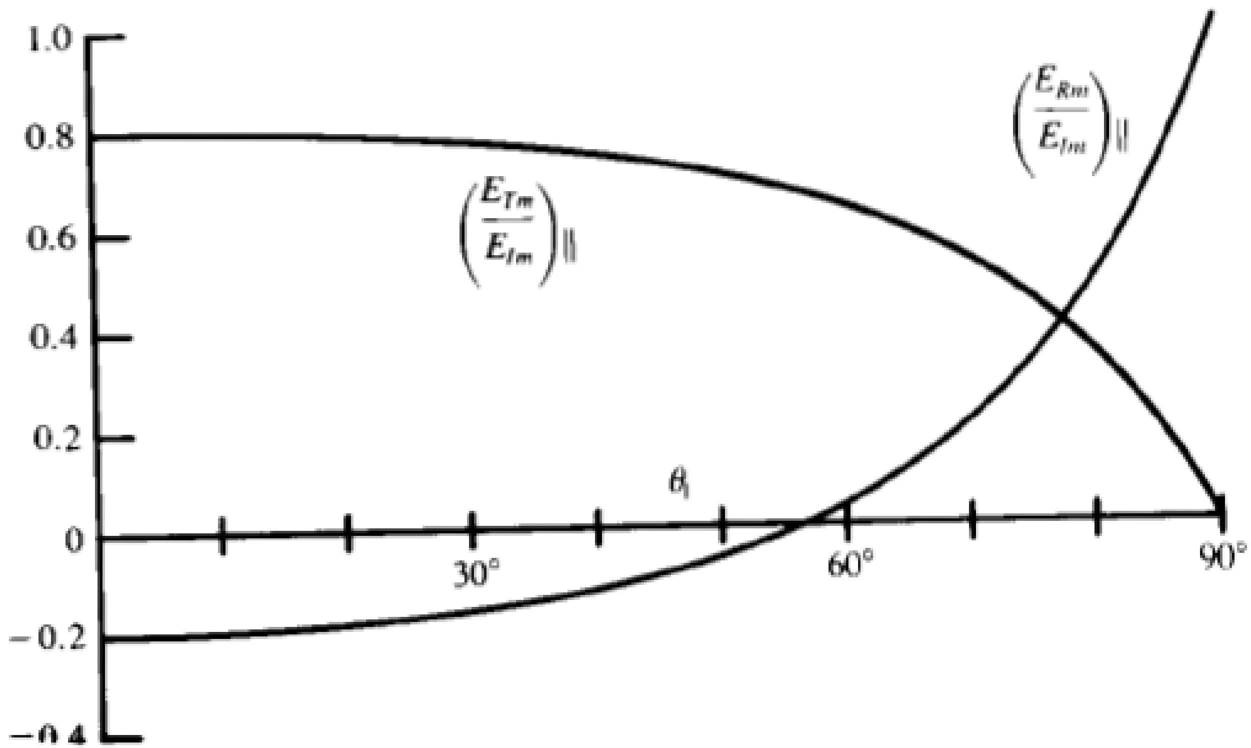
$$\sin\theta_T\cos\theta_T - \sin\theta_I\cos\theta_I > 0, \quad (30\text{-}43)$$

$$\sin 2\theta_T - \sin 2\theta_I > 0, \quad (30\text{-}44)$$

$$\sin(\theta_T - \theta_I)\cos(\theta_T + \theta_I) > 0. \quad (30\text{-}45)$$

$$\theta_T > \theta_I \quad \text{and} \quad \theta_T + \theta_I < \frac{\pi}{2} \quad (30\text{-}46)$$

$$\theta_T < \theta_I \quad \text{and} \quad \theta_T + \theta_I > \frac{\pi}{2}. \quad (30\text{-}47)$$

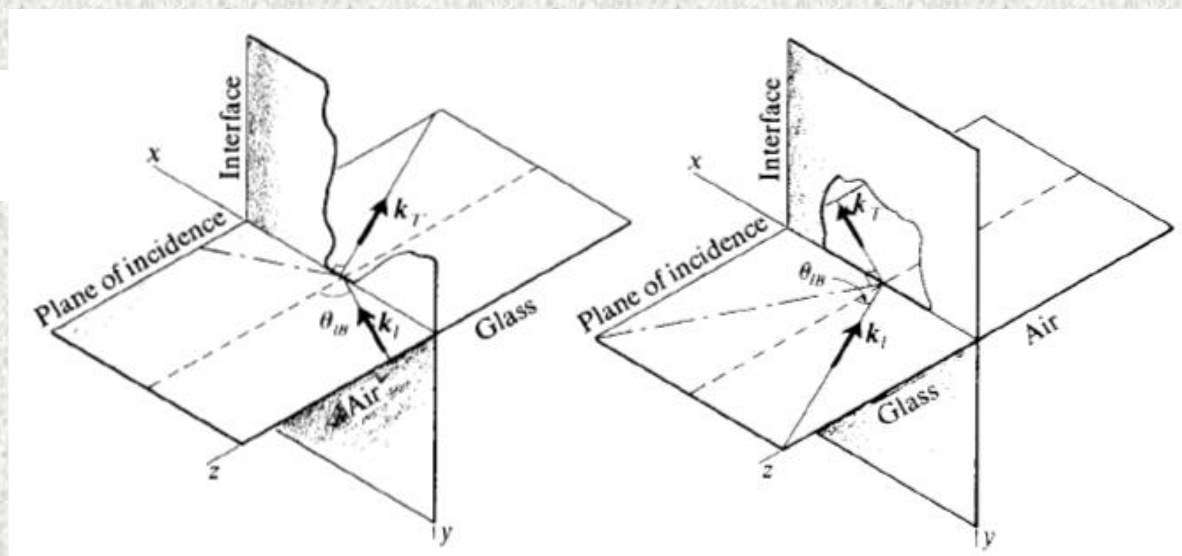


Reflection and refraction when $n_1/n_2 = 1/1.5$, with **E** *parallel* to the plane of incidence.

Brewster Angle

- When **E** is paralel to the plane of incidence, E_{rm} is either in phase or π radius out of phase with the incident wave depending on $\sin(\theta_T - \theta_I)\cos(\theta_T + \theta_I)$ is greater of less less than zero.
- There is no reflection when $\theta_T = \theta_I = 0$ or when $\theta_T + \theta_I = \pi/2$
- This angle of incidence is called Brewster angle. It also called polarizing angle
- At Brewster angle,

$$\frac{n_1}{n_2} = \frac{\sin\theta_T}{\sin\theta_{IB}} = \frac{\sin(\pi/2 - \theta_{IB})}{\sin\theta_{IB}} = \cot\theta_{IB}.$$



Coefficients of Reflection R and of Transmission T

- We exclude total reflection as well as reflection from conducting media.
- Setting μ_r=1, we find that

$$\mathscr{S}_{I,\text{av}} = \frac{1}{2}\left(\frac{\epsilon_1}{\mu_0}\right)^{1/2} E_{Im}^2 \hat{\boldsymbol{n}}_I, \qquad (30\text{-}51)$$

$$\mathscr{S}_{R,\text{av}} = \frac{1}{2}\left(\frac{\epsilon_1}{\mu_0}\right)^{1/2} E_{Rm}^2 \hat{\boldsymbol{n}}_R, \qquad (30\text{-}52)$$

$$\mathscr{S}_{T,\text{av}} = \frac{1}{2}\left(\frac{\epsilon_2}{\mu_0}\right)^{1/2} E_{Tm}^2 \hat{\boldsymbol{n}}_T, \qquad (30\text{-}53)$$

- Where $\overrightarrow{\boldsymbol{n}_I}$ is normal to a wave front of incident wave:

$$\hat{\boldsymbol{n}}_I = \frac{\boldsymbol{k}_I}{k_1}, \qquad (30\text{-}54)$$

- And similarly for $\boldsymbol{n}_R$ and $\boldsymbol{n}_T$

$$R = \left|\frac{\mathscr{S}_{R,\text{av}} \cdot \hat{\boldsymbol{n}}}{\mathscr{S}_{I,\text{av}} \cdot \hat{\mathbf{n}}}\right| = \frac{E_{Rm}^2}{E_{Im}^2}, \qquad (30\text{-}55)$$

$$T = \left|\frac{\mathscr{S}_{T,\text{av}} \cdot \hat{\boldsymbol{n}}}{\mathscr{S}_{I,\text{av}} \cdot \hat{\boldsymbol{n}}}\right| = \left(\frac{\epsilon_{r2}}{\epsilon_{r1}}\right)^{1/2} \frac{E_{Tm}^2}{E_{Im}^2} \frac{\cos\theta_T}{\cos\theta_I} = \frac{n_2 E_{Tm}^2 \cos\theta_T}{n_1 E_{Im}^2 \cos\theta_I}. \qquad (30\text{-}56)$$

- Then from Fresnel's equation fo nonconductors

$$R_\perp = \left[\frac{(n_1/n_2)\cos\theta_I - \cos\theta_T}{(n_1/n_2)\cos\theta_I + \cos\theta_T}\right]^2, \quad (30\text{-}57)$$

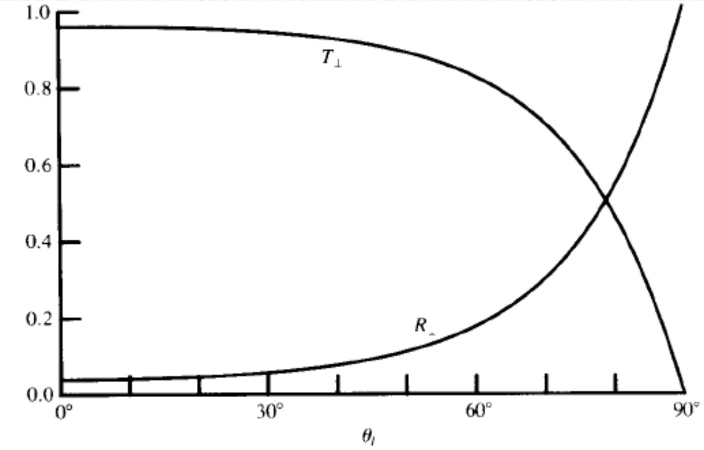
$$T_\perp = \frac{4(n_1/n_2)\cos\theta_I\cos\theta_T}{[(n_1/n_2)\cos\theta_I + \cos\theta_T]^2}, \quad (30\text{-}58)$$

$$R_\parallel = \left[\frac{-\cos\theta_I + (n_1/n_2)\cos\theta_T}{\cos\theta_I + (n_1/n_2)\cos\theta_T}\right]^2, \quad (30\text{-}59)$$

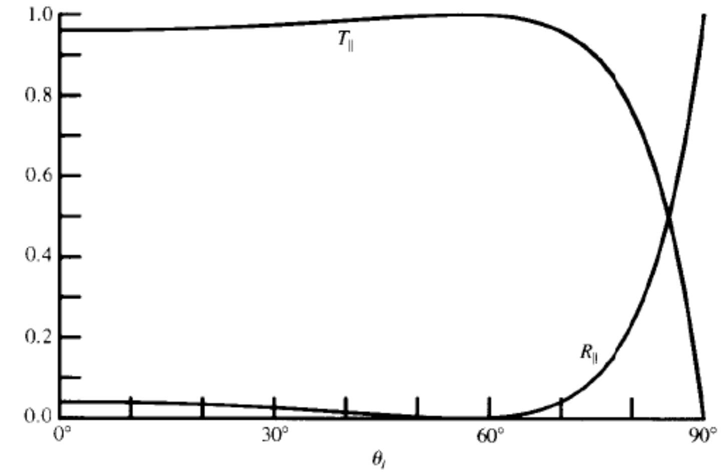
$$T_\parallel = \frac{4(n_1/n_2)\cos\theta_I\cos\theta_T}{[\cos\theta_I + (n_1/n_2)\cos\theta_T]^2}. \quad (30\text{-}60)$$

- In both instances *R+T=1*
- At Brewster Angle

$R_\parallel = 0$ and $T_\parallel = 1$,



The coefficients of reflection $R_\perp$ and of transmission $T_\perp$ as functions of the angle of incidence θ_I for $n_1/n_2 = 1/1.5$.



The coefficients of reflection $R_\parallel$ and of transmission $T_\parallel$ as functions of the angle of incidence θ_I for $n_1/n_2 = 1/1.5$. Note the Brewster angle at 56.3°.