EEE321 Electromagnetic Fileds and Waves

Prof. Dr. Hasan Hüseyin BALIK

(1st Week)

Outline

- Course Information and Policies
- Course Syllabus
- Vector Operators
- Coordinate Systems

Course Information
(see web for more details)

- Instructor: Prof. Dr. Hasan H. BALIK, hasanbalik@gmail.com, hasanbalik@aydin.edu.tr,

  www.hasanbalik.com
- Lecture Notes:

  http://www.hasanbalik.com/LectureNotes/EMFW/

  Book: Electromagnetic Fields and Waves, 3rd Edition (Paul Lorrain etc.)
- Grading: Midterm 20%, Popup Quiz (2) 20%, Assigment 20% and Final 40%

Course Syllabus-1

- Vector Operators and Coordinates Systems
- Electric Field
  - Coulumb's Law, Gauss's Law, The Equations of Poisson and Laplace, Charge Conservation, Conductors
  - Electric Multipoles, Energy, Capacitance, and Forces
  - Dielectric Materials
- Magnetic Fields
  - Magnetic Flux Density (**B**), Vector Potantial (**A**), Ampere's Circuital Law,
  - Magnetic Materials
  - Magnetic Fources on charges and Currents
  - The Faraday Induction Law, Magnetic Energy

Course Syllabus-2

- Electrimagnetic Waves
  - Maxwell Equations, Uniform Plane Waves in Free Space / Nonconductors / Conductors
  - Reflection and Refraction: The Basic Laws, Fresnel's Equations, Nonuniform Plane Waves, Total Reflection, Reflection and Refraction at the Surface of Good Conductor
- Antennas
  - The Electric Dipole Transmitting Antenna, The Half-Wave antenna, Antenna Arrays, The Magnetic Dipole Antenna

1.Vector Operators and Coordinate Systemd

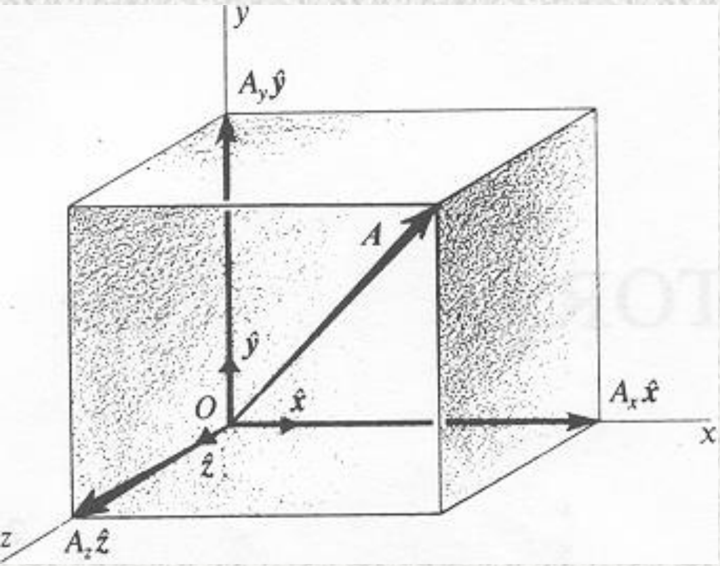
1.Outline

- Vector Algebra
- Invariance
- The Gradient ∇f
- Flux
- The Divergence $\nabla.\boldsymbol{B}$ and The Divergence Theorem
- The Line Integral $\int_a^b \boldsymbol{B}.\boldsymbol{dl}$. Conservative Fields
- The Curl $\nabla \boldsymbol{x}\boldsymbol{B}$ and Stokes's Theorem
- The Laplasian Operator ∇^2
- Orthogonal Curcilinear Coordinates

Introduction

- This chapter meant to help those students who are not yet proficient in the use of vector operators
- Mathematicaly a field is a function that describes a physical quantity at all points in the space
  - *Scalar Fields*: This quantity is specified by a single number at each point (Temperature, density, electric potetial)
  - *Vector Fields*:The physical quantitiy is a vector, specified by both number and direction (wind velocity, gravitational force)

Vector Algebra



- This figure shows a **A** vector and its three components A_x,A_y,A_z

$$\boldsymbol{A} = A_x\hat{\boldsymbol{x}} + A_y\hat{\boldsymbol{y}} + A_z\hat{\boldsymbol{z}}, \qquad \boldsymbol{B} = B_x\hat{\boldsymbol{x}} + B_y\hat{\boldsymbol{y}} + B_z\hat{\boldsymbol{z}}, \tag{1-1}$$

- Then the algebric operations

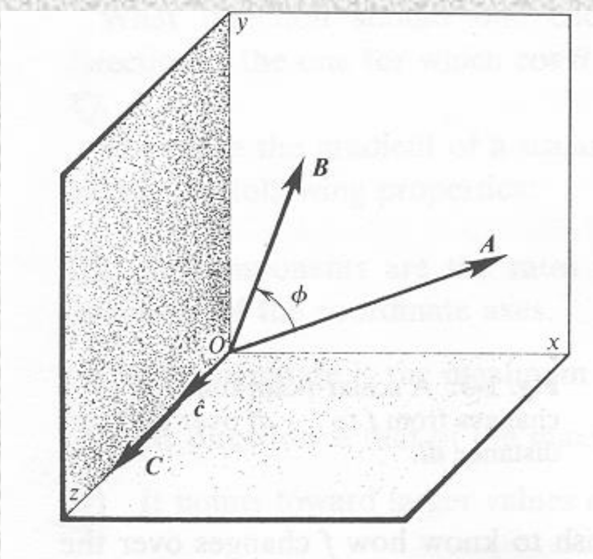
$$\boldsymbol{A} + \boldsymbol{B} = (A_x + B_x)\hat{\boldsymbol{x}} + (A_y + B_y)\hat{\boldsymbol{y}} + (A_z + B_z)\hat{\boldsymbol{z}}, \tag{1-2}$$

$$\boldsymbol{A} - \boldsymbol{B} = (A_x - B_x)\hat{\boldsymbol{x}} + (A_y - B_y)\hat{\boldsymbol{y}} + (A_z - B_z)\hat{\boldsymbol{z}}, \tag{1-3}$$

$$\boldsymbol{A} \cdot \boldsymbol{B} = A_xB_x + A_yB_y + A_zB_z = AB\cos\phi, \tag{1-4}$$

$$\boldsymbol{A} \times \boldsymbol{B} = \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = AB\sin\phi\hat{\boldsymbol{c}} = \boldsymbol{C}, \tag{1-5}$$

$$A = (A_x^2 + A_y^2 + A_z^2)^{1/2} \tag{1-6}$$

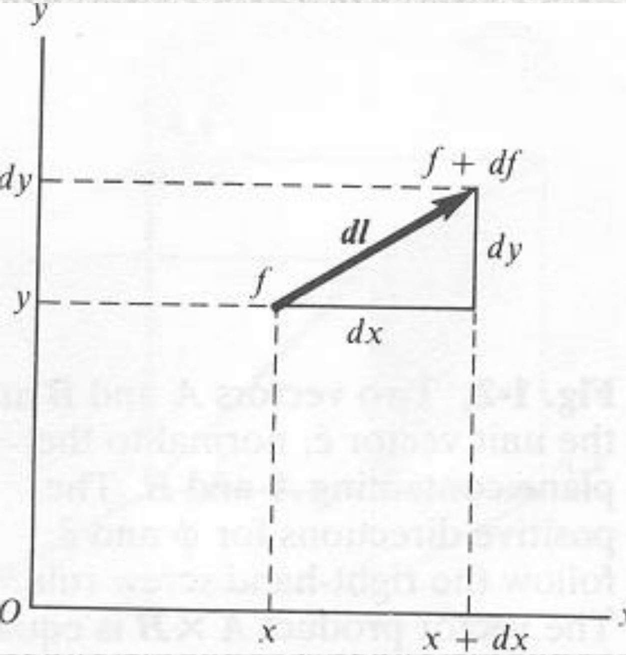


- The quantity $\boldsymbol{A}.\boldsymbol{B}$ is the scalar or dot product of **A** and **B**
- The quantity $\boldsymbol{AxB}$ is vector or cross product of of **A** and **B**

Invariance

- The quantities, that are independent of the choice of coordinate system, are said to be invariant
  - **A**, **B** and Φ, dot or cross products
- The quantities, that are dependent of the choice of coordinate system, are not invariant
  - Components of a vector

The Gradient ∇f-1



- Consider a scalar point function *f* that is continuous and differentable. We wish to know how *f* changes over the infinitesimal distancel *dl*

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz \qquad (1\text{-}7)$$

is scalar product of two vectors

$$\mathbf{dl} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}} \qquad (1\text{-}8)$$

and

$$\nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}. \qquad (1\text{-}9)$$

called *gradient of f.*

$$\nabla = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z} \qquad (1\text{-}10)$$

The symbol is read ‘del’

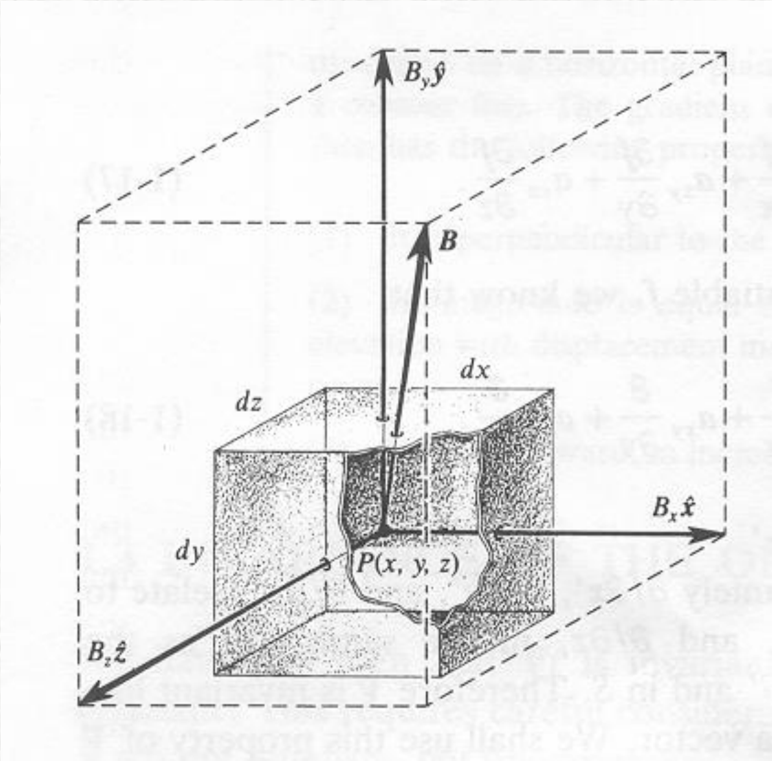
$$|\nabla f| = \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2\right]^{1/2}. \qquad (1\text{-}11)$$

$$df = \nabla f \cdot \mathbf{dl} = |\nabla f|\,|\mathbf{dl}|\cos\theta, \qquad (1\text{-}12)$$

The Gradient ∇f-2

- The gradient f a scalar function at a given point is a vector having the following properties
  - Its components are the rate of change of the function along the direction of the coordinate axes
  - Its magnetute is the maximum rate of change with distance
  - Its direction is that of the maximum rate of change with distance
  - It points toward larger values of the function

Flux



- It is often necessary to calculate flux of a vector quantity through a surface. The flux *dΦ of* ***B*** *through an infinitesimal surface* ***dA*** is

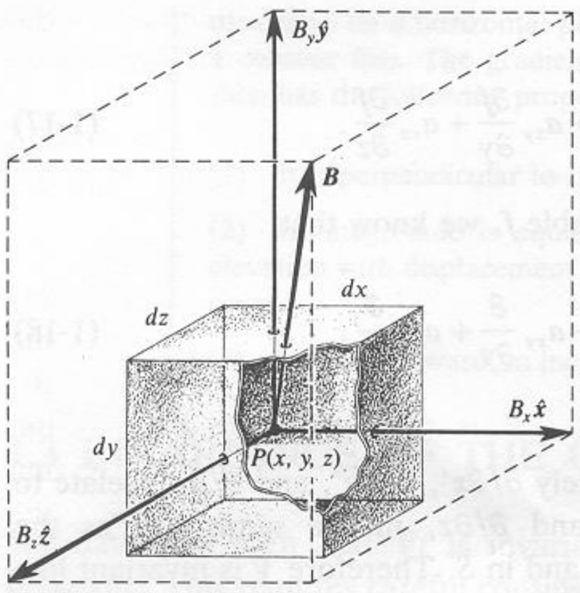
$$d\Phi = \boldsymbol{B} \cdot d\boldsymbol{\mathcal{A}}, \qquad (1\text{-}19)$$

- The vector ***dA*** is normal to surface.
- For a surface of finite area of *A*,

$$\Phi = \int_{\mathcal{A}} \boldsymbol{B} \cdot d\boldsymbol{\mathcal{A}}. \qquad (1\text{-}20)$$

- If surface is closed. The vector **dA** points outward

The Divergence $\nabla . B$



- The outward flux of a vector through a closed surface can be calculated either from the equations given at previous slide or as follows
- B_x, B_y, B_z are functions of x, y, z.
- The value B_x at the center of the right-hand face.

The outgoing flux for right-hand and left-

$$d\Phi_R = \left(B_x + \frac{\partial B_x}{\partial x}\frac{dx}{2}\right) dy\, dz, \qquad d\Phi_L = -\left(B_x - \frac{\partial B_x}{\partial x}\frac{dx}{2}\right) dy\, dz.$$

- The Total is

$$d\Phi_{tot} = \left(\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}\right) dv. \qquad (1\text{-}24)$$

$$\Phi_{tot} = \int_v \left(\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}\right) dv. \qquad (1\text{-}25)$$

- The divergence of **B** is

$$\nabla \cdot \boldsymbol{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}. \qquad (1\text{-}26)$$

The Divergence Theorem

- The total flux of a **B** is eaual to the surface integral of the normal outward components of **B**. If we donate by *A* the area of the surface bounding *v, the total flux is*

$$\Phi_{tot} = \int_{\mathcal{A}} \boldsymbol{B} \cdot d\boldsymbol{\mathcal{A}} = \int_{v} \left(\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} \right) dv = \int_{v} \boldsymbol{\nabla} \cdot \boldsymbol{B}\, dv. \qquad (1\text{-}27)$$

$$\int_{\mathcal{A}} \boldsymbol{B} \cdot d\boldsymbol{\mathcal{A}} = \int_{v} \boldsymbol{\nabla} \cdot \boldsymbol{B}\, dv. \qquad (1\text{-}28)$$

- This can be applied to any continuous differentiable vectors
- This is divergence theorem also called Green or Gauss's theorem

The Line Integral $\int_a^b B.\,dl$. Conservative Fields

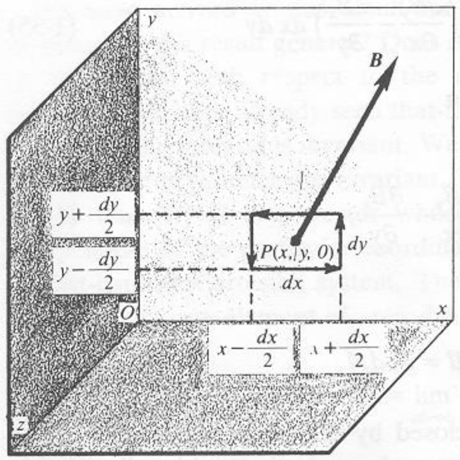
- The integrals

$$\int_a^b \boldsymbol{B}\cdot d\boldsymbol{l}, \qquad \int_a^b \boldsymbol{B}\times d\boldsymbol{l}, \qquad \text{and} \qquad \int_a^b f\,dl,$$

- Evaluated from a point a to the point b over some specified curve are examples of line integrals
- A vector field **B** is *conservative* if the line integral of ***B.dl*** around any closed curve is zero

$$\oint \boldsymbol{B}\cdot d\boldsymbol{l} = 0. \qquad (1\text{-}29)$$

The Curl $\nabla x B$-1



- For any givem field **B** and for a closed path situated in the *xy-plane*

$$\mathbf{B}\cdot d\mathbf{l} = B_x\,dx + B_y\,dy \qquad (1\text{-}30)$$

$$\oint \mathbf{B}\cdot d\mathbf{l} = \oint B_x\,dx + \oint B_y\,dy. \qquad (1\text{-}31)$$

- Now consider the infinitesimal path in the figure. There are two contributions.

$$\oint B_x\,dx = \left(B_x - \frac{\partial B_x}{\partial y}\frac{dy}{2}\right)dx - \left(B_x + \frac{\partial B_x}{\partial y}\frac{dy}{2}\right)dx. \qquad \oint B_x\,dx = -\frac{\partial B_x}{\partial y}\,dy\,dx. \qquad (1\text{-}33)$$

- Similarly

$$\oint B_y\,dy = \frac{\partial B_y}{\partial x}\,dx\,dy, \qquad (1\text{-}34)$$

- and

$$\oint \mathbf{B}\cdot d\mathbf{l} = \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)dx\,dy \qquad (1\text{-}35)$$

The Curl $\nabla x B$-2

- If we set

$$g_3 = \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}, \tag{1-36}$$

- Then

$$\oint \boldsymbol{B} \cdot d\boldsymbol{l} = g_3 \, d\mathcal{A}, \tag{1-37}$$

- Consider now g_3 and two symmetric quantities as the components of a vector

$$\nabla \times \boldsymbol{B} = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right)\hat{\boldsymbol{x}} + \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right)\hat{\boldsymbol{y}} + \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)\hat{\boldsymbol{z}}, \tag{1-38}$$

- Which may be written as

$$\nabla \times \boldsymbol{B} = \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} \tag{1-39}$$

- And $\oint \boldsymbol{B} \cdot d\boldsymbol{l} = (\nabla \times \boldsymbol{B}) \cdot d\boldsymbol{\mathcal{A}}.$ (1-41) $(\nabla \times \boldsymbol{B})_n = \lim_{\mathcal{A} \to 0} \frac{1}{\mathcal{A}} \oint_C \boldsymbol{B} \cdot d\boldsymbol{l}.$

Stokes'sTheorem

- Equation 1.41 is true only for a path so small that $\nabla x B$ is nearly constant over the surface ***dA*** bounded by the path.
- For any fiinite surface

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \int_{\mathscr{A}} (\nabla \times \mathbf{B}) \cdot d\mathscr{A}, \qquad (1\text{-}47)$$

- Where *A* is the area or any open surface bounded by the curvw *C*
- Equation 1.47 is called Skotes Theorem

The Laplacian Operator ∇^2

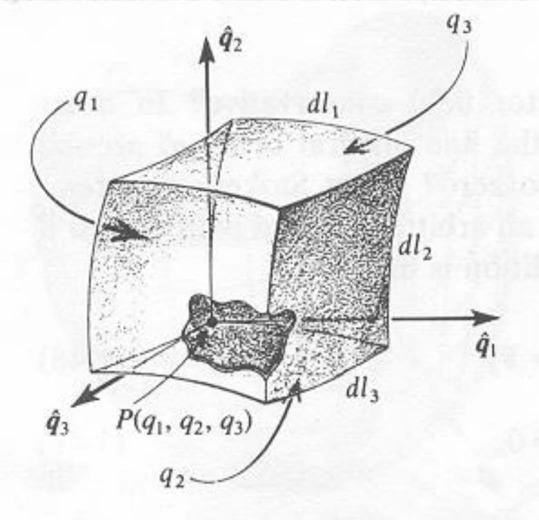
- The divergence of the gradient of *f* is the laplacian of *f*

$$\nabla \cdot \nabla f = \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}, \qquad (1\text{-}50)$$

- The laplacian can also be defined for a vector point-function **B**
- For Cartesian coordinates,:

$$\nabla^2 \mathbf{B} = \nabla^2 B_x \hat{\mathbf{x}} + \nabla^2 B_y \hat{\mathbf{y}} + \nabla^2 B_z \hat{\mathbf{z}}. \qquad (1\text{-}51)$$

Orthogonal Curcilinear Coordinates



- It is frequently inconvenient, because of the symmetries that exits in certain fields, to use Cartesian coordinates. Consider the equation

$$f(x, y, z) = q, \qquad (1\text{-}52)$$

- Consider three equations

$$f_1(x, y, z) = q_1, \qquad f_2(x, y, z) = q_2, \qquad f_3(x, y, z) = q_3 \qquad (1\text{-}53)$$

defining three families of surfaces that are mutualy orthogonal

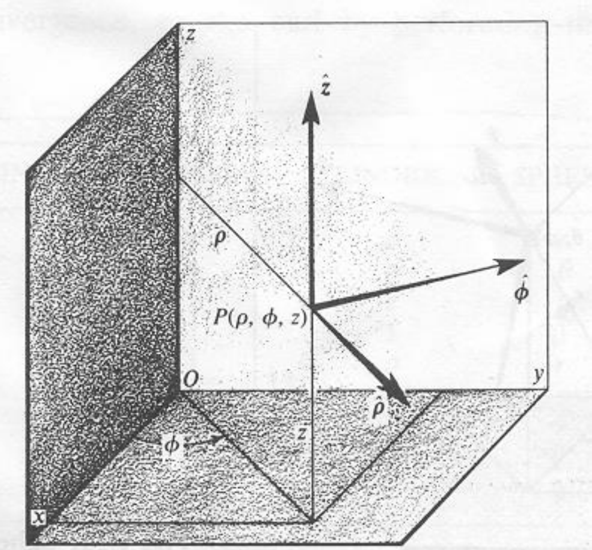
- Call dl_1 an element of length notmal to the surface q_1

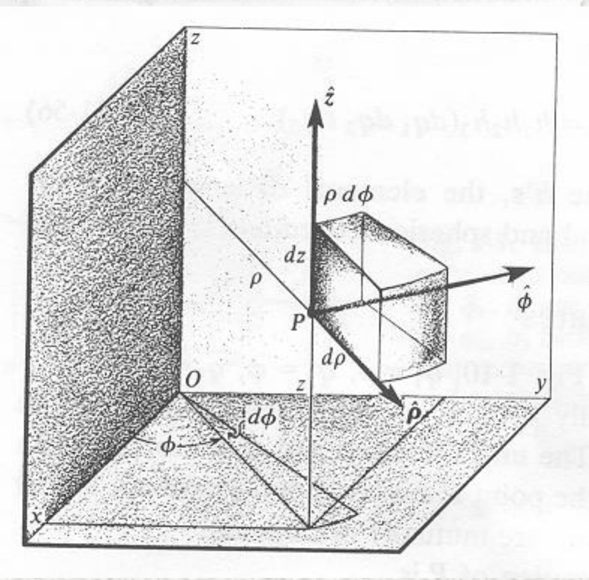
$$dl_1 = h_1\, dq_1, \qquad (1\text{-}54)$$

- Similarly $dl_2 = h_2\, dq_2$ and $dl_3 = h_3\, dq_3$. (1-55)

- The valume element is $dv = dl_1\, dl_2\, dl_3 = h_1 h_2 h_3 (dq_1\, dq_2\, dq_3).$

Cylindrical Coordinates





- In cylindrical coordinates $q_1=\rho$, $q_2=\Phi$ and $q_3=z$
- The vector that defines the point of P is

$$\mathbf{r} = \rho\hat{\boldsymbol{\rho}} + z\hat{\mathbf{z}}. \qquad (1\text{-}57)$$

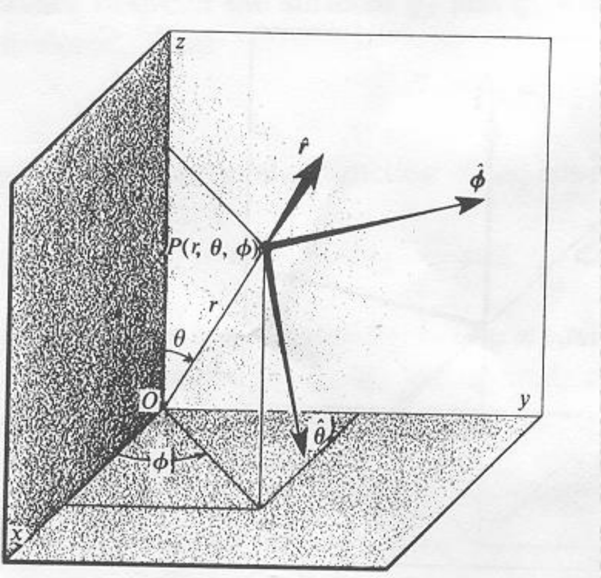
- The distance element is

$$d\mathbf{r} = d\rho\,\hat{\boldsymbol{\rho}} + \rho\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}. \qquad (1\text{-}58)$$

- The infinitesimal volume is

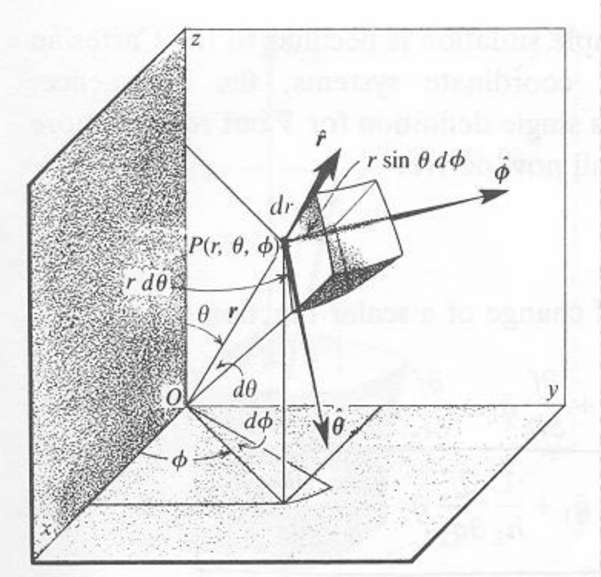
$$dv = \rho\,d\rho\,d\phi\,dz. \qquad (1\text{-}59)$$

Spherical Coordinates



- In sphericall coordinates $q_1=r$, $q_2=\theta$ and $q_3=\Phi$
- The distance element is

$$d\mathbf{r} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}. \qquad (1\text{-}60)$$



- The infinitesimal volume is

$$dv = r^2 \sin\theta\,dr\,d\theta\,d\phi. \qquad (1\text{-}61)$$

Correspondence Between Coordinates

CURVILINEAR	CARTESIAN	CYLINDRICAL	SPHERICAL
q_1	x	ρ	r
q_2	y	φ	θ
q_3	z	z	φ
h_1	1	1	1
h_2	1	ρ	r
h_3	1	1	$r \sin\theta$
$\hat{q}_1$	$\hat{x}$	$\hat{\rho}$	$\hat{r}$
$\hat{q}_2$	$\hat{y}$	$\hat{\varphi}$	$\hat{\theta}$
$\hat{q}_3$	$\hat{z}$	$\hat{z}$	$\hat{\varphi}$

The Gradient

- The gradient is the vector rate of change of a scalar function *f*

$$\nabla f = \frac{\partial f}{\partial l_1}\hat{q}_1 + \frac{\partial f}{\partial l_2}\hat{q}_2 + \frac{\partial f}{\partial l_3}\hat{q}_3 \qquad (1\text{-}62)$$

$$= \frac{1}{h_1}\frac{\partial f}{\partial q_1}\hat{q}_1 + \frac{1}{h_2}\frac{\partial f}{\partial q_2}\hat{q}_2 + \frac{1}{h_3}\frac{\partial f}{\partial q_3}\hat{q}_3. \qquad (1\text{-}63)$$

- For cylindrical coordinates

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}. \qquad (1\text{-}64)$$

- For spherical coordinates

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}. \qquad (1\text{-}65)$$

The Divergence

- The divergence for orthogonal curcilinear coordinates

$$\nabla \cdot \boldsymbol{B} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial q_1}(B_1 h_2 h_3) + \frac{\partial}{\partial q_2}(B_2 h_3 h_1) + \frac{\partial}{\partial q_3}(B_3 h_1 h_2)\right] \qquad (1\text{-}71)$$

- For cylindrical coordinates

$$\nabla \cdot \boldsymbol{B} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho B_\rho) + \frac{1}{\rho}\frac{\partial B_\phi}{\partial \phi} + \frac{\partial B_z}{\partial z} \qquad (1\text{-}72)$$

$$= \frac{B_\rho}{\rho} + \frac{\partial B_\rho}{\partial \rho} + \frac{1}{\rho}\frac{\partial B_\phi}{\partial \phi} + \frac{\partial B_z}{\partial z}. \qquad (1\text{-}73)$$

- For spherical coordinates

$$\nabla \cdot \boldsymbol{B} = \frac{1}{r^2 \sin\theta}\left[\frac{\partial}{\partial r}(B_r r^2 \sin\theta) + \frac{\partial}{\partial \theta}(B_\theta r \sin\theta) + \frac{\partial}{\partial \phi}(B_\phi r)\right] \qquad (1\text{-}74)$$

$$= \frac{2}{r}B_r + \frac{\partial B_r}{\partial r} + \frac{B_\theta}{r}\cot\theta + \frac{1}{r}\frac{\partial B_\theta}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial B_\phi}{\partial \phi}. \qquad (1\text{-}75)$$

The Curl

- The curl for orthogonal curcilinear coordinates

$$\nabla \times \boldsymbol{B} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \hat{\boldsymbol{q}}_1 & h_2 \hat{\boldsymbol{q}}_2 & h_3 \hat{\boldsymbol{q}}_3 \\ \frac{\partial}{\partial q_1} & \frac{\partial}{\partial q_2} & \frac{\partial}{\partial q_3} \\ h_1 B_1 & h_2 B_2 & h_3 B_3 \end{vmatrix} \qquad \text{(1-79)}$$

- For cylindrical coordinates

$$\nabla \times \boldsymbol{B} = \frac{1}{\rho} \begin{vmatrix} \hat{\boldsymbol{\rho}} & \rho \hat{\boldsymbol{\phi}} & \hat{\boldsymbol{z}} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ B_\rho & \rho B_\phi & B_z \end{vmatrix}, \qquad \text{(1-80)}$$

- For spherical coordinates

$$\nabla \times \boldsymbol{B} = \frac{1}{r^2 \sin \theta} \begin{vmatrix} \hat{\boldsymbol{r}} & r \hat{\boldsymbol{\theta}} & r \sin \theta \hat{\boldsymbol{\phi}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ B_r & r B_\theta & r \sin \theta B_\phi \end{vmatrix}. \qquad \text{(1-81)}$$

The Laplacian-1

- The laplace for scalar function *f*

$$\nabla^2 f = \nabla \cdot \nabla f \tag{1-82}$$

$$= \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial q_1}\left(\frac{h_2 h_3}{h_1}\frac{\partial f}{\partial q_1}\right) + \frac{\partial}{\partial q_2}\left(\frac{h_3 h_1}{h_2}\frac{\partial f}{\partial q_2}\right) + \frac{\partial}{\partial q_3}\left(\frac{h_1 h_2}{h_3}\frac{\partial f}{\partial q_3}\right)\right] \tag{1-83}$$

- For cylindrical coordinates

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2} \tag{1-84}$$

$$= \frac{1}{\rho}\frac{\partial f}{\partial \rho} + \frac{\partial^2 f}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}, \tag{1-85}$$

- For spherical coordinates

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial \phi^2} \tag{1-86}$$

$$= \frac{2}{r}\frac{\partial f}{\partial r} + \frac{\partial^2 f}{\partial r^2} + \frac{\cot\theta}{r^2}\frac{\partial f}{\partial \theta} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}, \tag{1-87}$$

The Laplacian-2

- For the laplace for vectoral function ***B**, the equation*

$$\nabla \times (\nabla \times \mathbf{B}) = \nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} \qquad (1\text{-}88)$$

- *is used. Then*

$$\nabla^2 \mathbf{B} = \nabla(\nabla \cdot \mathbf{B}) - \nabla \times (\nabla \times \mathbf{B}). \qquad (1\text{-}89)$$